Chapter 8

Potential Energy

▲ *A strobe photograph of a pole vaulter. During this process, several types of energy transformations occur. The two types of potential energy that we study in this chapter are evident in the photograph.* Gravitational potential energy *is associated with the change in vertical position of the vaulter relative to the Earth.* Elastic potential energy *is evident in the bending of the pole. (©Harold E. Edgerton/Courtesy of Palm Press, Inc.)*

CHAPTER OUTLINE

8.1 Potential Energy of a System

8.2 The Isolated System—Conservation of Mechanical Energy

8.3 Conservative and Nonconservative Forces

8.4 Changes in Mechanical Energy for Nonconservative Forces

8.5 Relationship Between Conservative Forces and Potential Energy

8.6 Energy Diagrams and Equilibrium of a System

In Chapter 7 we introduced the concepts of kinetic energy associated with the motion of members of a system and internal energy associated with the temperature of a system. In this chapter we introduce *potential energy*, the energy associated with the configuration of a system of objects that exert forces on each other.

The potential energy concept can be used only when dealing with a special class of forces called *conservative forces*. When only conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy. This balancing of the two forms of energy is known as the *principle of conservation of mechanical energy*.

Potential energy is present in the Universe in various forms, including gravitational, electromagnetic, chemical, and nuclear. Furthermore, one form of energy in a system can be converted to another. For example, when a system consists of an electric motor connected to a battery, the chemical energy in the battery is converted to kinetic energy as the shaft of the motor turns. The transformation of energy from one form to another is an essential part of the study of physics, engineering, chemistry, biology, geology, and astronomy.

8.1 Potential Energy of a System

In Chapter 7, we defined a system in general, but focused our attention primarily on single particles or objects under the influence of an external force. In this chapter, we consider systems of two or more particles or objects interacting via a force that is *internal* to the system. The kinetic energy of such a system is the algebraic sum of the kinetic energies of all members of the system. There may be systems, however, in which one object is so massive that it can be modeled as stationary and its kinetic energy can be neglected. For example, if we consider a ball–Earth system as the ball falls to the ground, the kinetic energy of the system can be considered as just the kinetic energy of the ball. The Earth moves so slowly in this process that we can ignore its kinetic energy. On the other hand, the kinetic energy of a system of two electrons must include the kinetic energies of both particles.

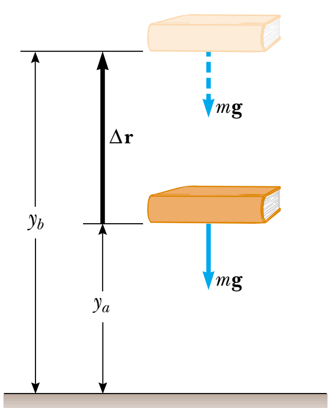


Figure 8.1 The work done by an external agent on the system of the book and the Earth as the book is lifted from a height y_a to a height y_b is equal to $mgy_b - mgy_a$.

Let us imagine a system consisting of a book and the Earth, interacting via the gravitational force. We do some work on the system by lifting the book slowly through a height $\Delta y = y_b - y_a$, as in Figure 8.1. According to our discussion of energy and energy transfer in Chapter 7, this work done on the system must appear as an increase in energy of the system. The book is at rest before we perform the work and is at rest after we perform the work. Thus, there is no change in the kinetic energy of the system. There is no reason why the temperature of the book or the Earth should change, so there is no increase in the internal energy of the system.

Because the energy change of the system is not in the form of kinetic energy or internal energy, it must appear as some other form of energy storage. After lifting the book, we could release it and let it fall back to the position y_a. Notice that the book (and, therefore, the system) will now have kinetic energy, and its source is in the work that was done in lifting the book. While the book was at the highest point, the energy of the system had the *potential* to become kinetic energy, but did not do so until the book was allowed to fall. Thus, we call the energy storage mechanism before we release the book **potential energy.** We will find that a potential energy can only be associated with specific types of forces. In this particular case, we are discussing **gravitational potential energy.**

Let us now derive an expression for the gravitational potential energy associated with an object at a given location above the surface of the Earth. Consider an external agent lifting an object of mass m from an initial height y_a above the ground to a final height y_b, as in Figure 8.1. We assume that the lifting is done slowly, with no acceleration, so that the lifting force can be modeled as being equal in magnitude to the weight of the object—the object is in equilibrium and moving at constant velocity. The work done by the external agent on the system (object and Earth) as the object undergoes this upward displacement is given by the product of the upward applied force $\mathbf{F}_{\text{app}}$ and the upward displacement $\Delta\mathbf{r} = \Delta y\hat{\mathbf{j}}$:

$$W = (\mathbf{F}_{\text{app}}) \cdot \Delta\mathbf{r} = (mg\hat{\mathbf{j}}) \cdot [(y_b - y_a)\hat{\mathbf{j}}] = mgy_b - mgy_a \tag{8.1}$$

Notice how similar this equation is to Equation 7.14 in the preceding chapter. In each equation, the work done on a system equals a difference between the final and initial values of a quantity. In Equation 7.14, the work represents a transfer of energy into the system, and the increase in energy of the system is kinetic in form. In Equation 8.1, the work represents a transfer of energy into the system, and the system energy appears in a different form, which we have called gravitational potential energy.

Thus, we can identify the quantity mgy as the gravitational potential energy U_g:

$$\boxed{U_g \equiv mgy} \tag{8.2}$$

Gravitational potential energy

The units of gravitational potential energy are joules, the same as those of work and kinetic energy. Potential energy, like work and kinetic energy, is a scalar quantity. Note that Equation 8.2 is valid only for objects near the surface of the Earth, where g is approximately constant.[1]

Using our definition of gravitational potential energy, Equation 8.1 can now be rewritten as

$$W = \Delta U_g \tag{8.3}$$

which mathematically describes the fact that the work done on the system in this situation appears as a change in the gravitational potential energy of the system.

The gravitational potential energy depends only on the vertical height of the object above the surface of the Earth. The same amount of work must be done on an object–Earth system whether the object is lifted vertically from the Earth or is pushed starting from the same point up a frictionless incline, ending up at the same height. This can be shown by calculating the work with a displacement having both vertical and horizontal components:

$$W = (\mathbf{F}_{\text{app}}) \cdot \Delta\mathbf{r} = (mg\hat{\mathbf{j}}) \cdot [(x_b - x_a)\hat{\mathbf{i}} + (y_b - y_a)\hat{\mathbf{j}}] = mgy_b - mgy_a$$

where there is no term involving x in the final result because $\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} = 0$.

In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, which is normally zero. The choice of reference configuration is completely arbitrary because the important quantity is the *difference* in potential energy and this difference is independent of the choice of reference configuration.

It is often convenient to choose as the reference configuration for zero potential energy the configuration in which an object is at the surface of the Earth, but this is not essential. Often, the statement of the problem suggests a convenient configuration to use.

[1] The assumption that g is constant is valid as long as the vertical displacement is small compared with the Earth's radius.

PITFALL PREVENTION

8.1 Potential Energy Belongs to a System

Potential energy is always associated with a *system* of two or more interacting objects. When a small object moves near the surface of the Earth under the influence of gravity, we may sometimes refer to the potential energy "associated with the object" rather than the more proper "associated with the system" because the Earth does not move significantly. We will not, however, refer to the potential energy "of the object" because this clearly ignores the role of the Earth.

Quick Quiz 8.1 Choose the correct answer. The gravitational potential energy of a system (a) is always positive (b) is always negative (c) can be negative or positive.

Quick Quiz 8.2 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the kinetic energy of the system, we (a) must include the kinetic energy of both the object and the Earth (b) can ignore the kinetic energy of the Earth because it is not part of the system (c) can ignore the kinetic energy of the Earth because the Earth is so massive compared to the object.

Quick Quiz 8.3 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the potential energy of the system, we identify the system as (a) both the object and the Earth (b) only the object (c) only the Earth.

Example 8.1 The Bowler and the Sore Toe

A bowling ball held by a careless bowler slips from the bowler's hands and drops on the bowler's toe. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the ball–Earth system as the ball falls. Repeat the calculation, using the top of the bowler's head as the origin of coordinates.

Solution First, we need to estimate a few values. A bowling ball has a mass of approximately 7 kg, and the top of a person's toe is about 0.03 m above the floor. Also, we shall assume the ball falls from a height of 0.5 m. Keeping nonsignificant digits until we finish the problem, we calculate the gravitational potential energy of the ball–Earth system just before the ball is released to be $U_i = mgy_i = (7\text{ kg})(9.80\text{ m/s}^2)(0.5\text{ m}) = 34.3\text{ J}$. A similar calculation for when the ball reaches his toe gives $U_f = mgy_f = (7\text{ kg})(9.80\text{ m/s}^2)(0.03\text{ m}) = 2.06\text{ J}$. So, the change in gravitational potential energy of the ball–Earth system is $\Delta U_g = U_f - U_i = -32.24\text{ J}$. We should probably keep only one digit because of the roughness of our estimates; thus, we estimate that the change in gravitational potential energy is -30 J. The system had 30 J of gravitational potential energy relative to the top of the toe before the ball began its fall.

When we use the bowler's head (which we estimate to be 1.50 m above the floor) as our origin of coordinates, we find that $U_i = mgy_i = (7\text{ kg})(9.80\text{ m/s}^2)(-1\text{ m}) = -68.6\text{ J}$ and $U_f = mgy_f = (7\text{ kg})(9.80\text{ m/s}^2)(-1.47\text{ m}) = -100.8\text{ J}$. The change in gravitational potential energy of the ball–Earth system is $\Delta U_g = U_f - U_i = -32.24\text{ J} \approx \boxed{-30\text{ J}}$. This is the same value as before, as it must be.

8.2 The Isolated System–Conservation of Mechanical Energy

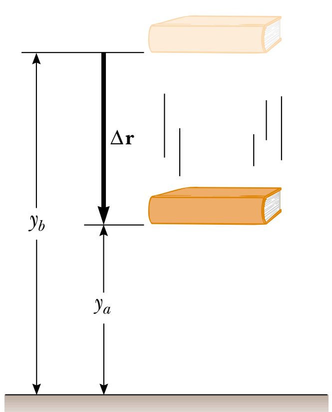


Figure 8.2 The work done by the gravitational force on the book as the book falls from y_b to a height y_a is equal to $mgy_b - mgy_a$.

The introduction of potential energy allows us to generate a powerful and universally applicable principle for solving problems that are difficult to solve with Newton's laws. Let us develop this new principle by thinking about the book–Earth system in Figure 8.1 again. After we have lifted the book, there is gravitational potential energy stored in the system, which we can calculate from the work done by the external agent on the system, using $W = \Delta U_g$.

Let us now shift our focus to the work done on the book alone by the gravitational force (Fig. 8.2) as the book falls back to its original height. As the book falls from y_b to y_a, the work done by the gravitational force on the book is

$$W_{\text{on book}} = (m\mathbf{g}) \cdot \Delta\mathbf{r} = (-mg\hat{\mathbf{j}}) \cdot [(y_a - y_b)\hat{\mathbf{j}}] = mgy_b - mgy_a \quad (8.4)$$

From the work–kinetic energy theorem of Chapter 7, the work done on the book is equal to the change in the kinetic energy of the book:

$$W_{\text{on book}} = \Delta K_{\text{book}}$$

Therefore, equating these two expressions for the work done on the book,

$$\Delta K_{\text{book}} = mgy_b - mgy_a \tag{8.5}$$

Now, let us relate each side of this equation to the *system* of the book and the Earth. For the right-hand side,

$$mgy_b - mgy_a = -(mgy_a - mgy_b) = -(U_f - U_i) = -\Delta U_g$$

where U_g is the gravitational potential energy of the system. For the left-hand side of Equation 8.5, because the book is the only part of the system that is moving, we see that $\Delta K_{\text{book}} = \Delta K$, where K is the kinetic energy of the system. Thus, with each side of Equation 8.5 replaced with its system equivalent, the equation becomes

$$\Delta K = -\Delta U_g \tag{8.6}$$

This equation can be manipulated to provide a very important general result for solving problems. First, we bring the change in potential energy to the left side of the equation:

$$\Delta K + \Delta U_g = 0 \tag{8.7}$$

On the left, we have a sum of changes of the energy stored in the system. The right hand is zero because there are no transfers of energy across the boundary of the system—the book–Earth system is *isolated* from the environment.

We define the sum of kinetic and potential energies as **mechanical energy:**

$$E_{\text{mech}} = K + U_g$$

We will encounter other types of potential energy besides gravitational later in the text, so we can write the general form of the definition for mechanical energy without a subscript on U:

$$\boxed{E_{\text{mech}} \equiv K + U} \tag{8.8}$$

Mechanical energy of a system

where U represents the total of *all* types of potential energy.

Let us now write the changes in energy in Equation 8.7 explicitly:

$$(K_f - K_i) + (U_f - U_i) = 0$$

$$\boxed{K_f + U_f = K_i + U_i} \tag{8.9}$$

The mechanical energy of an isolated, friction-free system is conserved.

For the gravitational situation that we have described, Equation 8.9 can be written as

$$\tfrac{1}{2}mv_f^2 + mgy_f = \tfrac{1}{2}mv_i^2 + mgy_i$$

As the book falls to the Earth, the book–Earth system loses potential energy and gains kinetic energy, such that the total of the two types of energy always remains constant.

Equation 8.9 is a statement of **conservation of mechanical energy** for an **isolated system.** An isolated system is one for which there are no energy transfers across the boundary. The energy in such a system is conserved—the sum of the kinetic and potential energies remains constant. (This statement assumes that no *nonconservative forces* act within the system; see Pitfall Prevention 8.2.)

Quick Quiz 8.4 In an isolated system, which of the following is a correct statement of the quantity that is conserved? (a) kinetic energy (b) potential energy (c) kinetic energy plus potential energy (d) both kinetic energy and potential energy.

PITFALL PREVENTION

8.2 Conditions on Equation 8.6

Equation 8.6 is true for only one of two categories of forces. These forces are called *conservative forces*, as discussed in the next section.

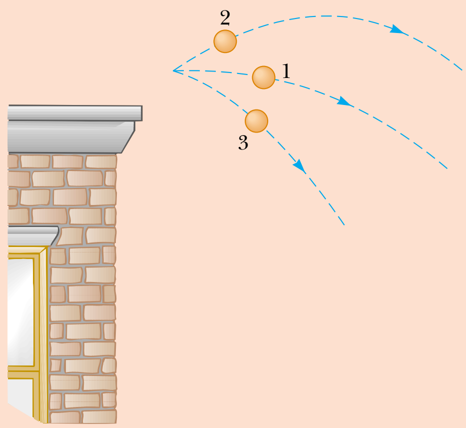
PITFALL PREVENTION

8.3 Mechanical Energy in an Isolated System

Equation 8.9 is not the only statement we can make for an isolated system. This describes conservation of *mechanical energy only* for the isolated system. We will see shortly how to include internal energy. In later chapters, we will generate new conservation statements (and associated equations) related to other conserved quantities.

Quick Quiz 8.5 A rock of mass m is dropped to the ground from a height h. A second rock, with mass $2m$, is dropped from the same height. When the second rock strikes the ground, its kinetic energy is (a) twice that of the first rock (b) four times that of the first rock (c) the same as that of the first rock (d) half as much as that of the first rock (e) impossible to determine.

Quick Quiz 8.6 Three identical balls are thrown from the top of a building, all with the same initial speed. The first is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal, as shown in Figure 8.3. Neglecting air resistance, rank the speeds of the balls at the instant each hits the ground.



Active Figure 8.3 (Quick Quiz 8.6) Three identical balls are thrown with the same initial speed from the top of a building.

At the Active Figures link at http://www.pse6.com, you can throw balls at different angles from the top of the building and compare the trajectories and the speeds as the balls hit the ground.

Elastic Potential Energy

We are familiar now with gravitational potential energy; let us explore a second type of potential energy. Consider a system consisting of a block plus a spring, as shown in Figure 8.4. The force that the spring exerts on the block is given by $F_s = -kx$. In the previous chapter, we learned that the work done by an external applied force F_{app} on a system consisting of a block connected to the spring is given by Equation 7.12:

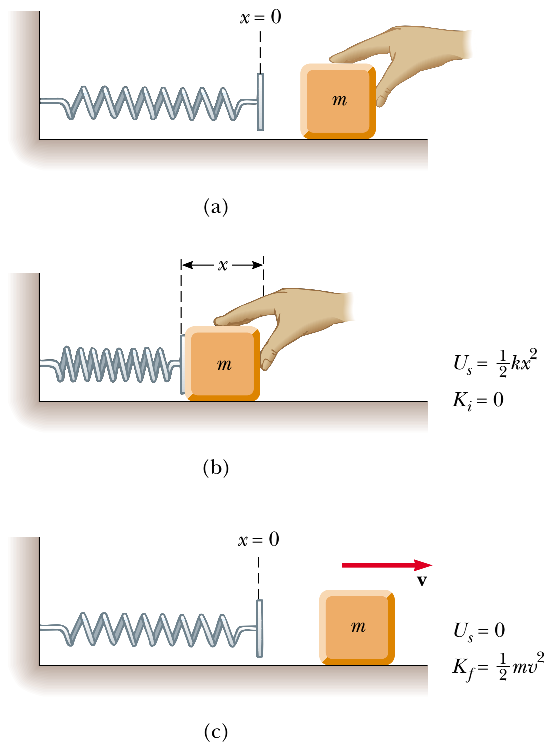
$$W_{F_{\text{app}}} = \tfrac{1}{2}kx_f^{\,2} - \tfrac{1}{2}kx_i^{\,2} \qquad (8.10)$$

In this situation, the initial and final x coordinates of the block are measured from its equilibrium position, $x = 0$. Again (as in the gravitational case), we see that the work done on the system is equal to the difference between the initial and final values of an expression related to the configuration of the system. The **elastic potential energy** function associated with the block–spring system is defined by

Elastic potential energy stored in a spring

$$U_s \equiv \tfrac{1}{2}kx^2 \qquad (8.11)$$

The elastic potential energy of the system can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position). To visualize this, consider Figure 8.4, which shows a spring on a frictionless, horizontal surface. When a block is pushed against the spring (Fig. 8.4b) and the spring is compressed a distance x, the elastic potential energy stored in the spring is $\tfrac{1}{2}kx^2$.



Active Figure 8.4 (a) An undeformed spring on a frictionless horizontal surface. (b) A block of mass m is pushed against the spring, compressing it a distance x. (c) When the block is released from rest, the elastic potential energy stored in the spring is transferred to the block in the form of kinetic energy.

At the Active Figures link at http://www.pse6.com, you can compress the spring by varying amounts and observe the effect on the block's speed.

When the block is released from rest, the spring exerts a force on the block and returns to its original length. The stored elastic potential energy is transformed into kinetic energy of the block (Fig. 8.4c).

The elastic potential energy stored in a spring is zero whenever the spring is undeformed ($x = 0$). Energy is stored in the spring only when the spring is either stretched or compressed. Furthermore, the elastic potential energy is a maximum when the spring has reached its maximum compression or extension (that is, when $|x|$ is a maximum). Finally, because the elastic potential energy is proportional to x^2, we see that U_s is always positive in a deformed spring.

Quick Quiz 8.7 A ball is connected to a light spring suspended vertically, as shown in Figure 8.5. When displaced downward from its equilibrium position and released, the ball oscillates up and down. In the system of *the ball, the spring, and the Earth,* what forms of energy are there during the motion? (a) kinetic and elastic potential (b) kinetic and gravitational potential (c) kinetic, elastic potential, and gravitational potential (d) elastic potential and gravitational potential.

Quick Quiz 8.8 Consider the situation in Quick Quiz 8.7 once again. In the system of *the ball and the spring,* what forms of energy are there during the motion? (a) kinetic and elastic potential (b) kinetic and gravitational potential (c) kinetic, elastic potential, and gravitational potential (d) elastic potential and gravitational potential.

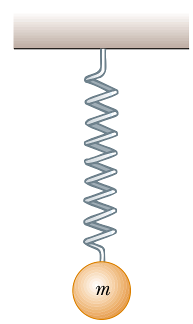


Figure 8.5 (Quick Quizzes 8.7 and 8.8) A ball connected to a massless spring suspended vertically. What forms of potential energy are associated with the system when the ball is displaced downward?

PROBLEM-SOLVING HINTS

Isolated Systems–Conservation of Mechanical Energy

We can solve many problems in physics using the principle of conservation of mechanical energy. You should incorporate the following procedure when you apply this principle:

- Define your isolated system, which may include two or more interacting particles, as well as springs or other structures in which elastic potential energy can be stored. Be sure to include all components of the system that exert forces on each other. Identify the initial and final configurations of the system.
- Identify configurations for zero potential energy (both gravitational and spring). If there is more than one force acting within the system, write an expression for the potential energy associated with each force.
- If friction or air resistance is present, mechanical energy of the system is not conserved and the techniques of Section 8.4 must be employed.
- If mechanical energy of the system is conserved, you can write the total energy $E_i = K_i + U_i$ for the initial configuration. Then, write an expression for the total energy $E_f = K_f + U_f$ for the final configuration that is of interest. Because mechanical energy is conserved, you can equate the two total energies and solve for the quantity that is unknown.

Example 8.2 Ball in Free Fall

Interactive

A ball of mass m is dropped from a height h above the ground, as shown in Figure 8.6.

(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.

Solution Figure 8.6 and our everyday experience with falling objects allow us to conceptualize the situation. While we can readily solve this problem with the techniques of Chapter 2, let us take an energy approach and categorize this as an energy problem for practice. To analyze the problem, we identify the system as the ball and the Earth. Because there is no air resistance and the system is isolated, we apply the principle of conservation of mechanical energy to the ball–Earth system.

At the instant the ball is released, its kinetic energy is $K_i = 0$ and the potential energy of the system is $U_i = mgh$. When the ball is at a distance y above the ground, its kinetic energy is $K_f = \frac{1}{2}mv_f^{\,2}$ and the potential energy relative to the ground is $U_f = mgy$. Applying Equation 8.9, we obtain

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}mv_f^{\,2} + mgy = 0 + mgh$$

$$v_f^{\,2} = 2g(h - y)$$

$$v_f = \boxed{\sqrt{2g(h-y)}}$$

The speed is always positive. If we had been asked to find the ball's velocity, we would use the negative value of the square root as the y component to indicate the downward motion.

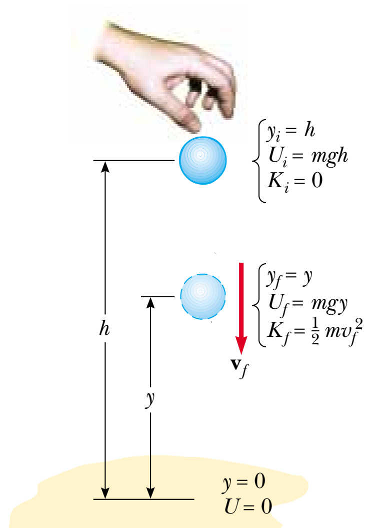


Figure 8.6 (Example 8.2) A ball is dropped from a height h above the ground. Initially, the total energy of the ball–Earth system is potential energy, equal to mgh relative to the ground. At the elevation y, the total energy is the sum of the kinetic and potential energies.

(B) Determine the speed of the ball at y if at the instant of release it already has an initial upward speed v_i at the initial altitude h.

Solution In this case, the initial energy includes kinetic energy equal to $\frac{1}{2}mv_i^{\,2}$ and Equation 8.9 gives

$$\tfrac{1}{2}mv_f^{\,2} + mgy = \tfrac{1}{2}mv_i^{\,2} + mgh$$

$$v_f^{\,2} = v_i^{\,2} + 2g(h - y)$$

$$v_f = \boxed{\sqrt{v_i^{\,2} + 2g(h - y)}}$$

Note that this result is consistent with the expression $v_{yf}^{\,2} = v_{yi}^{\,2} - 2g(y_f - y_i)$ from kinematics, where $y_i = h$. Furthermore, this result is valid even if the initial velocity is at an angle to the horizontal (Quick Quiz 8.6) for two reasons: (1) energy is a scalar, and the kinetic energy depends only on the magnitude of the velocity; and (2) the change in the gravitational potential energy depends only on the change in position in the vertical direction.

What If? **What if the initial velocity $\mathbf{v}_i$ in part (B) were downward? How would this affect the speed of the ball at position y?**

Answer We might be tempted to claim that throwing it downward would result in it having a higher speed at y than if we threw it upward. Conservation of mechanical energy, however, depends on kinetic and potential energies, which are scalars. Thus, the direction of the initial velocity vector has no bearing on the final speed.

Compare the effect of upward, downward, and zero initial velocities at the Interactive Worked Example link at **http://www.pse6.com.**

Example 8.3 The Pendulum

A pendulum consists of a sphere of mass m attached to a light cord of length L, as shown in Figure 8.7. The sphere is released from rest at point Ⓐ when the cord makes an angle θ_A with the vertical, and the pivot at P is frictionless.

(A) Find the speed of the sphere when it is at the lowest point Ⓑ.

Solution The only force that does work on the sphere is the gravitational force. (The force applied by the cord is always perpendicular to each element of the displacement and hence does no work.) Because the pendulum–Earth system is isolated, the energy of the system is conserved. As the pendulum swings, continuous transformation between potential and kinetic energy occurs. At the instant the pendulum is released, the energy of the system is entirely potential energy. At point Ⓑ the pendulum has kinetic energy, but the system has lost some potential energy. At Ⓒ the system has regained its initial potential energy, and the kinetic energy of the pendulum is again zero.

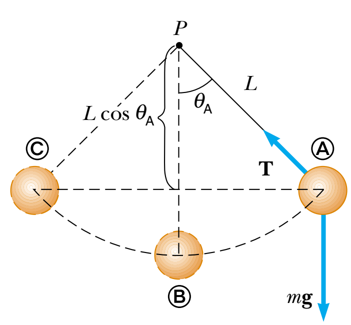


Figure 8.7 (Example 8.3) If the sphere is released from rest at the angle θ_A, it will never swing above this position during its motion. At the start of the motion, when the sphere is at position Ⓐ, the energy of the sphere–Earth system is entirely potential. This initial potential energy is transformed into kinetic energy when the sphere is at the lowest elevation Ⓑ. As the sphere continues to move along the arc, the energy again becomes entirely potential energy when the sphere is at Ⓒ.

If we measure the y coordinates of the sphere from the center of rotation, then $y_A = -L\cos\theta_A$ and $y_B = -L$. Therefore, $U_A = -mgL\cos\theta_A$ and $U_B = -mgL$.

Applying the principle of conservation of mechanical energy to the system gives

$$K_B + U_B = K_A + U_A$$

$$\tfrac{1}{2}mv_B^{\,2} - mgL = 0 - mgL\cos\theta_A$$

$$(1) \qquad \boxed{v_B = \sqrt{2gL(1 - \cos\theta_A)}}$$

(B) What is the tension T_B in the cord at Ⓑ?

Solution Because the tension force does no work, it does not enter into an energy equation, and we cannot determine the tension using the energy method. To find T_B, we can apply Newton's second law to the radial direction. First, recall that the centripetal acceleration of a particle moving in a circle is equal to v^2/r directed toward the center of rotation. Because $r = L$ in this example, Newton's second law gives

$$(2) \qquad \sum F_r = mg - T_B = ma_r = -m\frac{v_B^{\,2}}{L}$$

Substituting Equation (1) into Equation (2) gives the tension at point Ⓑ as a function of θ_A:

$$(3) \qquad T_B = mg + 2mg(1 - \cos\theta_A) = \boxed{mg(3 - 2\cos\theta_A)}$$

From Equation (2) we see that the tension at Ⓑ is greater than the weight of the sphere. Furthermore, Equation (3) gives the expected result that $T_B = mg$ when the initial angle $\theta_A = 0$. Note also that part (A) of this example is categorized as an energy problem while part (B) is categorized as a Newton's second law problem.

Example 8.4 A Grand Entrance Interactive

You are designing an apparatus to support an actor of mass 65 kg who is to "fly" down to the stage during the performance of a play. You attach the actor's harness to a 130-kg sandbag by means of a lightweight steel cable running smoothly over two frictionless pulleys, as in Figure 8.8a. You need 3.0 m of cable between the harness and the nearest pulley so that the pulley can be hidden behind a curtain. For the apparatus to work successfully, the sandbag must

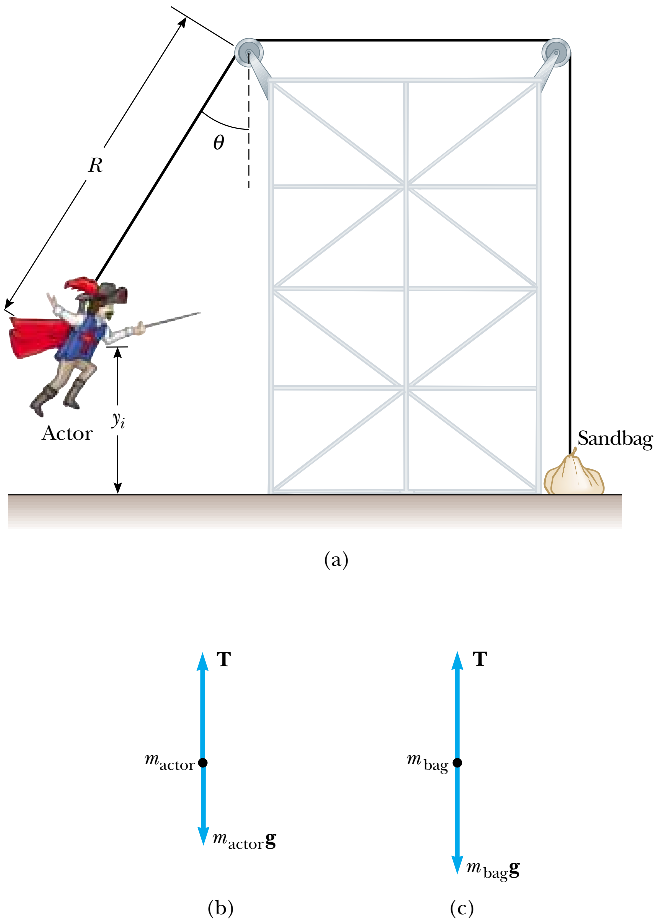


Figure 8.8 (Example 8.4) (a) An actor uses some clever staging to make his entrance. (b) Free-body diagram for the actor at the bottom of the circular path. (c) Free-body diagram for the sandbag.

never lift above the floor as the actor swings from above the stage to the floor. Let us call the initial angle that the actor's cable makes with the vertical θ. What is the maximum value θ can have before the sandbag lifts off the floor?

Solution We must use several concepts to solve this problem. To conceptualize, imagine what happens as the actor approaches the bottom of the swing. At the bottom, the cable is vertical and must support his weight as well as provide centripetal acceleration of his body in the upward direction. At this point, the tension in the cable is the highest and the sandbag is most likely to lift off the floor. Looking first at the swinging of the actor from the initial point to the lowest point, we categorize this as an energy problem involving an isolated system—the actor and the Earth. We use the principle of conservation of mechanical energy for the system to find the actor's speed as he arrives at the floor as a function of the initial angle θ and the radius R of the circular path through which he swings.

Applying conservation of mechanical energy to the actor–Earth system gives

$$K_f + U_f = K_i + U_i$$

$$(1) \qquad \tfrac{1}{2} m_{\text{actor}} v_f^2 + 0 = 0 + m_{\text{actor}} g y_i$$

where y_i is the initial height of the actor above the floor and v_f is the speed of the actor at the instant before he lands. (Note that $K_i = 0$ because he starts from rest and that $U_f = 0$ because we define the configuration of the actor at the floor as having a gravitational potential energy of zero.) From the geometry in Figure 8.8a, and noting that $y_f = 0$, we see that $y_i = R - R\cos\theta = R(1 - \cos\theta)$. Using this relationship in Equation (1), we obtain

$$(2) \qquad v_f^2 = 2gR(1 - \cos\theta)$$

Next, we focus on the instant the actor is at the lowest point. Because the tension in the cable is transferred as a force applied to the sandbag, we categorize the situation at this instant as a Newton's second law problem. We apply Newton's second law to the actor at the bottom of his path, using the free-body diagram in Figure 8.8b as a guide:

$$\sum F_y = T - m_{\text{actor}} g = m_{\text{actor}} \frac{v_f^2}{R}$$

$$(3) \qquad T = m_{\text{actor}} g + m_{\text{actor}} \frac{v_f^2}{R}$$

Finally, we note that the sandbag lifts off the floor when the upward force exerted on it by the cable exceeds the gravitational force acting on it; the normal force is zero when this happens. Thus, when we focus our attention on the sandbag, we categorize this part of the situation as another Newton's second law problem. A force T of the magnitude given by Equation (3) is transmitted by the cable to the sandbag. If the sandbag is to be just lifted off the floor, the normal force on it becomes zero and we require that $T = m_{\text{bag}} g$, as in Figure 8.8c. Using this condition together with Equations (2) and (3), we find that

$$m_{\text{bag}} g = m_{\text{actor}} g + m_{\text{actor}} \frac{2gR(1 - \cos\theta)}{R}$$

Solving for $\cos\theta$ and substituting in the given parameters, we obtain

$$\cos\theta = \frac{3m_{\text{actor}} - m_{\text{bag}}}{2m_{\text{actor}}} = \frac{3(65\text{ kg}) - 130\text{ kg}}{2(65\text{ kg})} = 0.50$$

$$\theta = 60°$$

Note that we had to combine techniques from different areas of our study—energy and Newton's second law. Furthermore, we see that the length R of the cable from the actor's harness to the leftmost pulley did not appear in the final algebraic equation. Thus, the final answer is independent of R.

What If? What if a stagehand locates the sandbag so that the cable from the sandbag to the right-hand pulley in Figure 8.8a is not vertical but makes an angle ϕ with the vertical? If the actor swings from the angle found in the solution above, will the sandbag lift off the floor? Assume that the length R remains the same.

Answer In this situation, the gravitational force acting on the sandbag is no longer parallel to the cable. Thus, only a component of the force in the cable acts against the gravitational force, and the vertical resultant of this force component and the gravitational force should be downward. As a result, there should be a nonzero normal force to balance this resultant, and the sandbag should *not* lift off the floor.

If the sandbag is in equilibrium in the y direction and the normal force from the floor goes to zero, Newton's second law gives us $T\cos\phi = m_{\text{bag}}g$. In this case, Equation (3) gives

$$\frac{m_{\text{bag}}g}{\cos\phi} = m_{\text{actor}}g + m_{\text{actor}}\frac{v_f^2}{R}$$

Substituting for v_f from Equation (2) gives

$$\frac{m_{\text{bag}}g}{\cos\phi} = m_{\text{actor}}g + m_{\text{actor}}\frac{2gR(1-\cos\theta)}{R}$$

Solving for $\cos\theta$, we have

$$(4) \qquad \cos\theta = \frac{3m_{\text{actor}} - \dfrac{m_{\text{bag}}}{\cos\phi}}{2m_{\text{actor}}}$$

For $\phi = 0$, which is the situation in Figure 8.8a, $\cos\phi = 1$. For nonzero values of ϕ, the term $\cos\phi$ is smaller than 1. This makes the numerator of the fraction in Equation (4) smaller, which makes the angle θ larger. Thus, the sandbag remains on the floor if the actor swings from a larger angle. If he swings from the original angle, the sandbag remains on the floor. For example, suppose $\phi = 10°$. Then, Equation (4) gives

$$\cos\theta = \frac{3(65\text{ kg}) - \dfrac{130\text{ kg}}{\cos 10°}}{2(65\text{ kg})} = 0.48 \longrightarrow \theta = 61°$$

Thus, if he swings from 60°, he is swinging from an angle below the new maximum allowed angle, and the sandbag remains on the floor.

One factor we have not addressed is the friction force between the sandbag and the floor. If this is not large enough, the sandbag may break free and start to slide horizontally as the actor reaches some point in his swing. This will cause the length R to increase, and the actor may have a frightening moment as he begins to drop in addition to swinging!

Let the actor fly or crash without injury to people at the Interactive Worked Example link at* http://www.pse6.com*. You may choose to include the effect of friction between the sandbag and the floor.

Example 8.5 The Spring-Loaded Popgun

The launching mechanism of a toy gun consists of a spring of unknown spring constant (Fig. 8.9a). When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing.

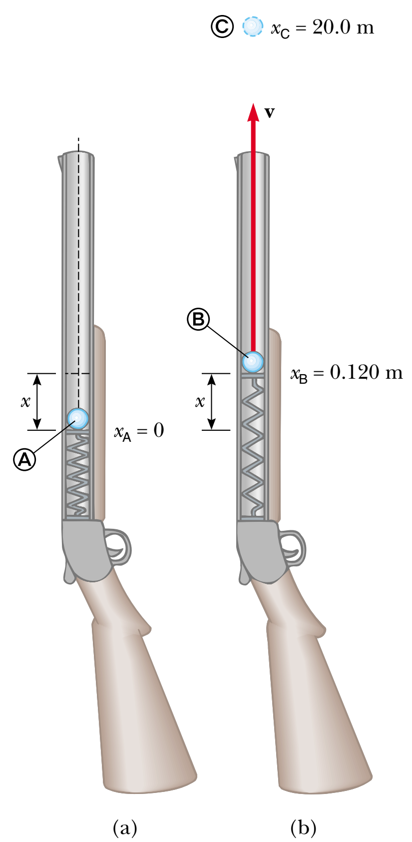


Figure 8.9 (Example 8.5) A spring-loaded popgun.

(A) Neglecting all resistive forces, determine the spring constant.

Solution Because the projectile starts from rest, its initial kinetic energy is zero. If we take the zero configuration for the gravitational potential energy of the projectile–spring–Earth system to be when the projectile is at the lowest position x_A, then the initial gravitational potential energy of the system also is zero. The mechanical energy of this system is conserved because the system is isolated.

Initially, the only mechanical energy in the system is the elastic potential energy stored in the spring of the gun, $U_{sA} = \frac{1}{2}kx^2$, where the compression of the spring is $x = 0.120$ m. The projectile rises to a maximum height $x_C = h = 20.0$ m, and so the final gravitational potential energy of the system when the projectile reaches its peak is mgh. The final kinetic energy of the projectile is zero, and the final elastic potential energy stored in the spring is zero. Because the mechanical energy of the system is conserved, we find that

$$E_C = E_A$$

$$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$$

$$0 + mgh + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

$$k = \frac{2mgh}{x^2} = \frac{2(0.035\,0\text{ kg})(9.80\text{ m/s}^2)(20.0\text{ m})}{(0.120\text{ m})^2}$$

$$= \boxed{953\text{ N/m}}$$

(B) Find the speed of the projectile as it moves through the equilibrium position of the spring (where $x_B = 0.120\,\text{m}$) as shown in Figure 8.9b.

Solution As already noted, the only mechanical energy in the system at Ⓐ is the elastic potential energy $\frac{1}{2}kx^2$. The total energy of the system as the projectile moves through the equilibrium position of the spring includes the kinetic energy of the projectile $\frac{1}{2}mv_B{}^2$ and the gravitational potential energy mgx_B of the system. Hence, the principle of conservation of mechanical energy in this case gives

$$E_B = E_A$$

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

$$\tfrac{1}{2}mv_B{}^2 + mgx_B + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

Solving for v_B gives

$$v_B = \sqrt{\frac{kx^2}{m} - 2gx_B}$$

$$= \sqrt{\frac{(953\text{ N/m})(0.120\text{ m})^2}{(0.0350\text{ kg})} - 2(9.80\text{ m/s}^2)(0.120\text{ m})}$$

$$= \boxed{19.7\text{ m/s}}$$

8.3 Conservative and Nonconservative Forces

As an object moves downward near the surface of the Earth, the work done by the gravitational force on the object does not depend on whether it falls vertically or slides down a sloping incline. All that matters is the change in the object's elevation. However, the energy loss due to friction on that incline depends on the distance the object slides. In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider the energy loss due to friction forces. We can use this varying dependence on path to classify forces as either conservative or nonconservative.

Of the two forces just mentioned, the gravitational force is conservative and the friction force is nonconservative.

Conservative Forces

Conservative forces have these two equivalent properties:

Properties of a conservative force

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

The gravitational force is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another. As we learned in the preceding section, the work done by the gravitational force on an object moving between any two points near the Earth's surface is $W_g = mgy_i - mgy_f$. From this equation, we see that W_g depends only on the initial and final y coordinates of the object and hence is independent of the path. Furthermore, W_g is zero when the object moves over any closed path (where $y_i = y_f$).

For the case of the object–spring system, the work W_s done by the spring force is given by $W_s = \frac{1}{2}kx_i{}^2 - \frac{1}{2}kx_f{}^2$ (Eq. 7.11). Again, we see that the spring force is conservative because W_s depends only on the initial and final x coordinates of the object and is zero for any closed path.

We can associate a potential energy for a system with any conservative force acting between members of the system and can do this only for conservative forces. In the previous section, the potential energy associated with the gravitational force was defined as $U_g \equiv mgy$. In general, the work W_c done by a conservative force on an object that is a member of a system as the object moves from one position to another is equal to the initial value of the potential energy of the system minus the final value:

$$W_c = U_i - U_f = -\Delta U \tag{8.12}$$

PITFALL PREVENTION

8.4 Similar Equation Warning

Compare Equation 8.12 to Equation 8.3. These equations are similar except for the negative sign, which is a common source of confusion. Equation 8.3 tells us that the work done *by an outside agent* on a system causes an increase in the potential energy of the system (with no change in the kinetic or internal energy). Equation 8.12 states that work done *on a component of a system by a conservative force internal to an isolated system* causes a decrease in the potential energy of the system (with a corresponding increase in kinetic energy).

This equation should look familiar to you. It is the general form of the equation for work done by the gravitational force (Eq. 8.4) as an object moves relative to the Earth and that for the work done by the spring force (Eq. 7.11) as the extension of the spring changes.

Nonconservative Forces

A force is **nonconservative** if it does not satisfy properties 1 and 2 for conservative forces. Nonconservative forces acting within a system cause a *change* in the mechanical energy E_{mech} of the system. We have defined mechanical energy as the sum of the kinetic and all potential energies. For example, if a book is sent sliding on a horizontal surface that is not frictionless, the force of kinetic friction reduces the book's kinetic energy. As the book slows down, its kinetic energy decreases. As a result of the friction force, the temperatures of the book and surface increase. The type of energy associated with temperature is internal energy, which we introduced in Chapter 7. Only part of the book's kinetic energy is transformed to internal energy in the book. The rest appears as internal energy in the surface. (When you trip and fall while running across a gymnasium floor, not only does the skin on your knees warm up, so does the floor!) Because the force of kinetic friction transforms the mechanical energy of a system into internal energy, it is a nonconservative force.

As an example of the path dependence of the work, consider Figure 8.10. Suppose you displace a book between two points on a table. If the book is displaced in a straight line along the blue path between points Ⓐ and Ⓑ in Figure 8.10, you do a certain amount of work against the kinetic friction force to keep the book moving at a constant speed. Now, imagine that you push the book along the brown semicircular path in Figure 8.10. You perform more work against friction along this longer path than along the straight path. The work done depends on the path, so the friction force cannot be conservative.

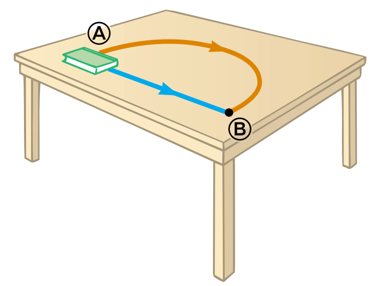


Figure 8.10 The work done against the force of kinetic friction depends on the path taken as the book is moved from Ⓐ to Ⓑ. The work is greater along the red path than along the blue path.

8.4 Changes in Mechanical Energy for Nonconservative Forces

As we have seen, if the forces acting on objects within a system are conservative, then the mechanical energy of the system is conserved. However, if some of the forces acting on objects within the system are not conservative, then the mechanical energy of the system changes.

Consider the book sliding across the surface in the preceding section. As the book moves through a distance d, the only force that does work on it is the force of kinetic friction. This force causes a decrease in the kinetic energy of the book. This decrease was calculated in Chapter 7, leading to Equation 7.20, which we repeat here:

$$\Delta K = -f_k d \tag{8.13}$$

Suppose, however, that the book is part of a system that also exhibits a change in potential energy. In this case, $-f_k d$ is the amount by which the *mechanical* energy of the system changes because of the force of kinetic friction. For example, if the book moves on an incline that is not frictionless, there is a change in both the kinetic energy and the gravitational potential energy of the book–Earth system. Consequently,

$$\Delta E_{mech} = \Delta K + \Delta U_g = -f_k d$$

In general, if a friction force acts within a system,

$$\boxed{\Delta E_{mech} = \Delta K + \Delta U = -f_k d} \tag{8.14}$$

Change in mechanical energy of a system due to friction within the system

where ΔU is the change in *all* forms of potential energy. Notice that Equation 8.14 reduces to Equation 8.9 if the friction force is zero.

Quick Quiz 8.9 A block of mass m is projected across a horizontal surface with an initial speed v. It slides until it stops due to the friction force between the block and the surface. The same block is now projected across the horizontal surface with an initial speed $2v$. When the block has come to rest, how does the distance from the projection point compare to that in the first case? (a) It is the same. (b) It is twice as large. (c) It is four times as large. (d) The relationship cannot be determined.

Quick Quiz 8.10 A block of mass m is projected across a horizontal surface with an initial speed v. It slides until it stops due to the friction force between the block and the surface. The surface is now tilted at 30°, and the block is projected up the surface with the same initial speed v. Assume that the friction force remains the same as when the block was sliding on the horizontal surface. When the block comes to rest momentarily, how does the decrease in mechanical energy of the block–surface–Earth system compare to that when the block slid over the horizontal surface? (a) It is the same. (b) It is larger. (c) It is smaller. (d) The relationship cannot be determined.

PROBLEM-SOLVING HINTS

Isolated Systems–Nonconservative Forces

You should incorporate the following procedure when you apply energy methods to a system in which nonconservative forces are acting:

- Follow the procedure in the first three bullets of the Problem-Solving Hints in Section 8.2. If nonconservative forces act within the system, the third bullet should tell you to use the techniques of this section.
- Write expressions for the total initial and total final mechanical energies of the system. The difference between the total final mechanical energy and the total initial mechanical energy equals the change in mechanical energy of the system due to friction.

Example 8.6 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0°, as shown in Figure 8.11. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.

Solution Because $v_i = 0$, the initial kinetic energy of the crate–Earth system when the crate is at the top of the ramp is zero. If the y coordinate is measured from the bottom of the ramp (the final position of the crate, for which the gravitational potential energy of the system is zero) with the upward direction being positive, then $y_i = 0.500\,\text{m}$. Therefore, the total mechanical energy of the system when the crate is at the top is all potential energy:

$$E_i = K_i + U_i = 0 + U_i = mgy_i$$
$$= (3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) = 14.7\text{ J}$$

When the crate reaches the bottom of the ramp, the potential energy of the system is zero because the elevation of

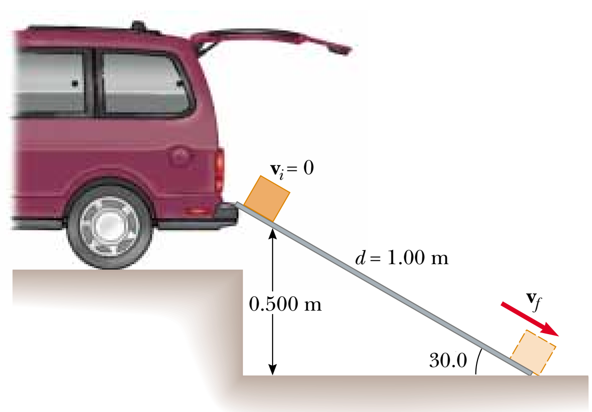


Figure 8.11 (Example 8.6) A crate slides down a ramp under the influence of gravity. The potential energy decreases while the kinetic energy increases.

the crate is $y_f = 0$. Therefore, the total mechanical energy of the system when the crate reaches the bottom is all kinetic energy:

$$E_f = K_f + U_f = \tfrac{1}{2} m v_f^2 + 0$$

We cannot say that $E_i = E_f$ because a nonconservative force reduces the mechanical energy of the system. In this case, Equation 8.14 gives $\Delta E_{\text{mech}} = -f_k d$, where d is the distance the crate moves along the ramp. (Remember that the forces normal to the ramp do no work on the crate because they are perpendicular to the displacement.) With $f_k = 5.00\text{ N}$ and $d = 1.00$ m, we have

$$(1) \qquad -f_k d = (-5.00\text{ N})(1.00\text{ m}) = -5.00\text{ J}$$

Applying Equation 8.14 gives

$$E_f - E_i = \tfrac{1}{2} m v_f^2 - m g y_i = -f_k d$$

$$(2) \qquad \tfrac{1}{2} m v_f^2 = 14.7\text{ J} - 5.00\text{ J} = 9.70\text{ J}$$

$$v_f^2 = \frac{19.4\text{ J}}{3.00\text{ kg}} = 6.47\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{2.54\text{ m/s}}$$

What If? **A cautious worker decides that the speed of the crate when it arrives at the bottom of the ramp may be so large that its contents may be damaged. Therefore, he replaces the ramp with a longer one such that the new ramp makes an angle of 25° with the ground. Does this new ramp reduce the speed of the crate as it reaches the ground?**

Answer Because the ramp is longer, the friction force will act over a longer distance and transform more of the mechanical energy into internal energy. This reduces the kinetic energy of the crate, and we expect a lower speed as it reaches the ground.

We can find the length d of the new ramp as follows:

$$\sin 25° = \frac{0.500\text{ m}}{d} \longrightarrow d = \frac{0.500\text{ m}}{\sin 25°} = 1.18\text{ m}$$

Now, Equation (1) becomes

$$-f_k d = (-5.00\text{ N})(1.18\text{ m}) = -5.90\text{ J}$$

and Equation (2) becomes

$$\tfrac{1}{2} m v_f^2 = 14.7\text{ J} - 5.90\text{ J} = 8.80\text{ J}$$

leading to

$$v_f = 2.42\text{ m/s}$$

The final speed is indeed lower than in the higher-angle case.

Example 8.7 Motion on a Curved Track

A child of mass m rides on an irregularly curved slide of height $h = 2.00$ m, as shown in Figure 8.12. The child starts from rest at the top.

(A) Determine his speed at the bottom, assuming no friction is present.

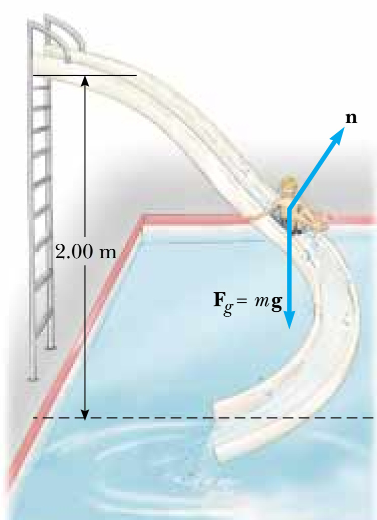


Figure 8.12 (Example 8.7) If the slide is frictionless, the speed of the child at the bottom depends only on the height of the slide.

Solution Although you have no experience on totally frictionless surfaces, you can conceptualize that your speed at the bottom of a frictionless ramp would be greater than in the situation in which friction acts. If we tried to solve this problem with Newton's laws, we would have a difficult time because the acceleration of the child continuously varies in direction due to the irregular shape of the slide. The child–Earth system is isolated and frictionless, however, so we can categorize this as a conservation of energy problem and search for a solution using the energy approach. (Note that the normal force **n** does no work on the child because this force is always perpendicular to each element of the displacement.) To analyze the situation, we measure the y coordinate in the upward direction from the bottom of the slide so that $y_i = h$, $y_f = 0$, and we obtain

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2} m v_f^2 + 0 = 0 + mgh$$

$$v_f = \sqrt{2gh}$$

Note that the result is the same as it would be had the child fallen vertically through a distance h! In this example, $h = 2.00$ m, giving

$$v_f = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(2.00\text{ m})} = \boxed{6.26\text{ m/s}}$$

(B) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that $v_f = 3.00$ m/s and $m = 20.0$ kg.

Solution We categorize this case, with friction, as a problem in which a nonconservative force acts. Hence, mechanical energy is not conserved, and we must use Equation 8.14 to find the loss of mechanical energy due to friction:

$$\begin{aligned}\Delta E_{\text{mech}} &= (K_f + U_f) - (K_i + U_i) \\ &= (\tfrac{1}{2}mv_f^{\,2} + 0) - (0 + mgh) = \tfrac{1}{2}mv_f^{\,2} - mgh \\ &= \tfrac{1}{2}(20.0\text{ kg})(3.00\text{ m/s})^2 \\ &\quad - (20.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) \\ &= \boxed{-302\text{ J}}\end{aligned}$$

Again, ΔE_{mech} is negative because friction is reducing the mechanical energy of the system. (The final mechanical energy is less than the initial mechanical energy.)

What If? Suppose you were asked to find the coefficient of friction μ_k for the child on the slide. Could you do this?

Answer We can argue that the same final speed could be obtained by having the child travel down a short slide with large friction or a long slide with less friction. Thus, there does not seem to be enough information in the problem to determine the coefficient of friction.

The energy loss of 302 J must be equal to the product of the friction force and the length of the slide:

$$-f_k d = -302\text{ J}$$

We can also argue that the friction force can be expressed as $\mu_k n$, where n is the magnitude of the normal force. Thus,

$$\mu_k n d = 302\text{ J}$$

If we try to evaluate the coefficient of friction from this relationship, we run into two problems. First, there is no single value of the normal force n unless the angle of the slide relative to the horizontal remains fixed. Even if the angle were fixed, we do not know its value. The second problem is that we do not have information about the length d of the slide. Thus, we cannot find the coefficient of friction from the information given.

Example 8.8 Let's Go Skiing!

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure 8.13. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop?

Solution The system is the skier plus the Earth, and we choose as our configuration of zero potential energy that in which the skier is at the bottom of the incline. While the skier is on the frictionless incline, the mechanical energy of the system remains constant, and we find, as we did in Example 8.7, that

$$v_{\text{B}} = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(20.0\text{ m})} = 19.8\text{ m/s}$$

Now we apply Equation 8.14 as the skier moves along the rough horizontal surface from Ⓑ to Ⓒ. The change in mechanical energy along the horizontal surface is

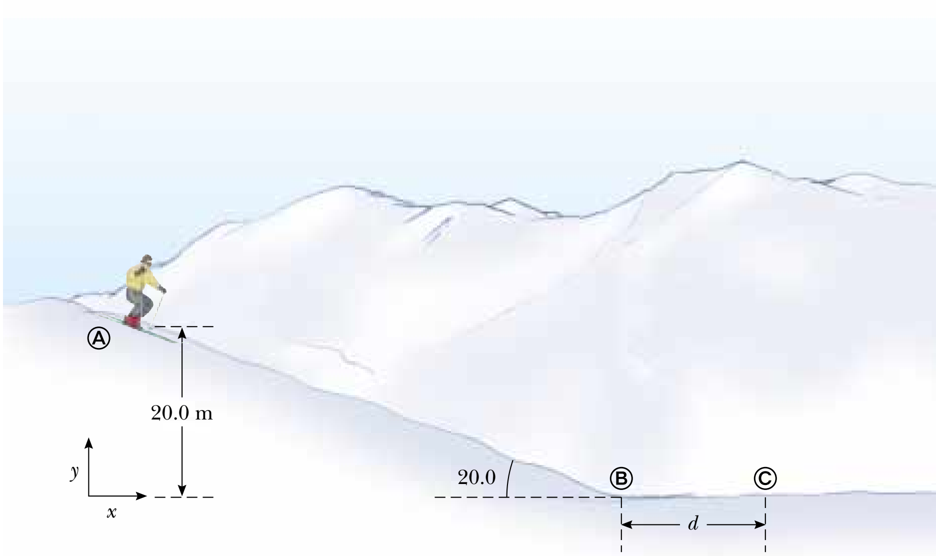


Figure 8.13 (Example 8.8) The skier slides down the slope and onto a level surface, stopping after a distance d from the bottom of the hill.

$\Delta E_{mech} = -f_k d$, where d is the horizontal distance traveled by the skier.

To find the distance the skier travels before coming to rest, we take $K_C = 0$. With $v_B = 19.8$ m/s and the friction force given by $f_k = \mu_k n = \mu_k mg$, we obtain

$$\Delta E_{mech} = E_C - E_B = -\mu_k mgd$$

$$(K_C + U_C) - (K_B + U_B) = (0 + 0) - (\tfrac{1}{2}mv_B^2 + 0) = -\mu_k mgd$$

$$d = \frac{v_B^2}{2\mu_k g} = \frac{(19.8 \text{ m/s})^2}{2(0.210)(9.80 \text{ m/s}^2)} = \boxed{95.2 \text{ m}}$$

Example 8.9 Block–Spring Collision

A block having a mass of 0.80 kg is given an initial velocity $v_A = 1.2$ m/s to the right and collides with a spring of negligible mass and force constant $k = 50$ N/m, as shown in Figure 8.14.

(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

Solution Our system in this example consists of the block and spring. All motion takes place in a horizontal plane, so we do not need to consider changes in gravitational potential energy. Before the collision, when the block is at Ⓐ, it has kinetic energy and the spring is uncompressed, so the elastic potential energy stored in the spring is zero. Thus, the total mechanical energy of the system before the collision is just $\frac{1}{2}mv_A^2$. After the collision, when the block is at Ⓒ, the spring is fully compressed; now the block is at rest and so has zero kinetic energy, while the energy stored in the spring has its maximum value $\frac{1}{2}kx^2 = \frac{1}{2}kx_{max}^2$, where the origin of coordinates $x = 0$ is chosen to be the equilibrium position of the spring and x_{max} is the maximum compression of the spring, which in this case happens to be x_C. The total mechanical energy of the system is conserved because no nonconservative forces act on objects within the system.

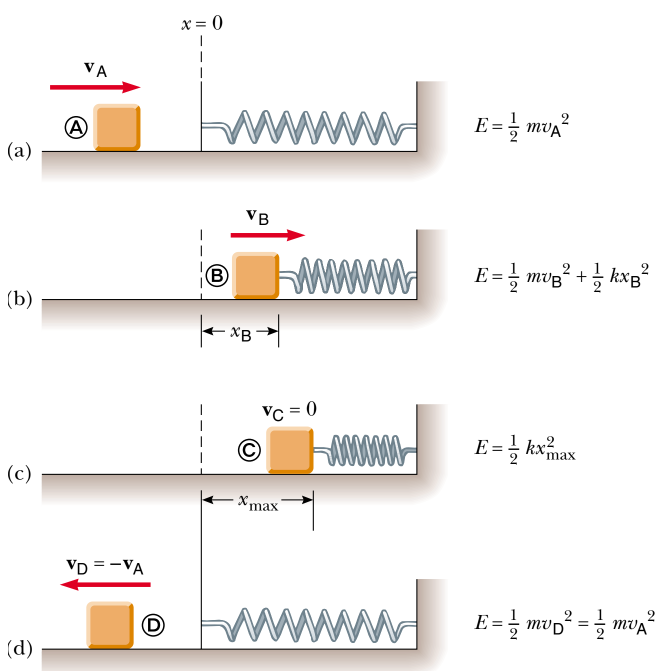


Figure 8.14 (Example 8.9) A block sliding on a smooth, horizontal surface collides with a light spring. (a) Initially the mechanical energy is all kinetic energy. (b) The mechanical energy is the sum of the kinetic energy of the block and the elastic potential energy in the spring. (c) The energy is entirely potential energy. (d) The energy is transformed back to the kinetic energy of the block. The total energy of the system remains constant throughout the motion.

Because the mechanical energy of the system is conserved, the kinetic energy of the block before the collision equals the maximum potential energy stored in the fully compressed spring:

$$E_C = E_A$$

$$K_C + U_{sC} = K_A + U_{sA}$$

$$0 + \tfrac{1}{2}kx_{max}^2 = \tfrac{1}{2}mv_A^2 + 0$$

$$x_{max} = \sqrt{\frac{m}{k}}\,v_A = \sqrt{\frac{0.80 \text{ kg}}{50 \text{ N/m}}}\,(1.2 \text{ m/s}) = \boxed{0.15 \text{ m}}$$

(B) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_A = 1.2$ m/s, what is the maximum compression x_C in the spring?

Solution In this case, the mechanical energy of the system is *not* conserved because a friction force acts on the block. The magnitude of the friction force is

$$f_k = \mu_k n = \mu_k mg = 0.50(0.80 \text{ kg})(9.80 \text{ m/s}^2) = 3.92 \text{ N}$$

Therefore, the change in the mechanical energy of the system due to friction as the block is displaced from the equilibrium position of the spring (where we have set our origin) to x_C is

$$\Delta E_{mech} = -f_k x_C = (-3.92x_C)$$

Substituting this into Equation 8.14 gives

$$\Delta E_{mech} = E_f - E_i = (0 + \tfrac{1}{2}kx_C^2) - (\tfrac{1}{2}mv_A^2 + 0) = -f_k x_C$$

$$\tfrac{1}{2}(50)x_C^2 - \tfrac{1}{2}(0.80)(1.2)^2 = -3.92x_C$$

$$25x_C^2 + 3.92x_C - 0.576 = 0$$

Solving the quadratic equation for x_C gives $x_C = 0.092$ m and $x_C = -0.25$ m. The physically meaningful root is $x_C = 0.092$ m. The negative root does not apply to this situation because the block must be to the right of the origin (positive value of x) when it comes to rest. Note that the value of 0.092 m is less than the distance obtained in the frictionless case of part (A). This result is what we expect because friction retards the motion of the system.

Example 8.10 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.15. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

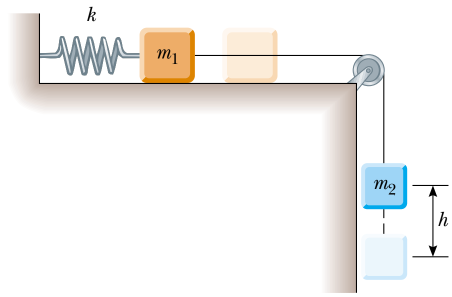


Figure 8.15 (Example 8.10) As the hanging block moves from its highest elevation to its lowest, the system loses gravitational potential energy but gains elastic potential energy in the spring. Some mechanical energy is lost because of friction between the sliding block and the surface.

Solution The key word *rest* appears twice in the problem statement. This suggests that the configurations associated with rest are good candidates for the initial and final configurations because the kinetic energy of the system is zero for these configurations. (Also note that because we are concerned only with the beginning and ending points of the motion, we do not need to label events with circled letters as we did in the previous two examples. Simply using i and f is sufficient to keep track of the situation.) In this situation, the system consists of the two blocks, the spring, and the Earth. We need to consider two forms of potential energy: gravitational and elastic. Because the initial and final kinetic energies of the system are zero, $\Delta K = 0$, and we can write

$$(1) \qquad \Delta E_{\text{mech}} = \Delta U_g + \Delta U_s$$

where $\Delta U_g = U_{gf} - U_{gi}$ is the change in the system's gravitational potential energy and $\Delta U_s = U_{sf} - U_{si}$ is the change in the system's elastic potential energy. As the hanging block falls a distance h, the horizontally moving block moves the same distance h to the right. Therefore, using Equation 8.14, we find that the loss in mechanical energy in the system due to friction between the horizontally sliding block and the surface is

$$(2) \qquad \Delta E_{\text{mech}} = -f_k h = -\mu_k m_1 g h$$

The change in the gravitational potential energy of the system is associated with only the falling block because the vertical coordinate of the horizontally sliding block does not change. Therefore, we obtain

$$(3) \qquad \Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

where the coordinates have been measured from the lowest position of the falling block.

The change in the elastic potential energy of the system is that stored in the spring:

$$(4) \qquad \Delta U_s = U_{sf} - U_{si} = \tfrac{1}{2}kh^2 - 0$$

Substituting Equations (2), (3), and (4) into Equation (1) gives

$$-\mu_k m_1 g h = -m_2 g h + \tfrac{1}{2}kh^2$$

$$\mu_k = \frac{m_2 g - \frac{1}{2}kh}{m_1 g}$$

This setup represents a way of measuring the coefficient of kinetic friction between an object and some surface. As you can see from the problem, sometimes it is easier to work with the changes in the various types of energy rather than the actual values. For example, if we wanted to calculate the numerical value of the gravitational potential energy associated with the horizontally sliding block, we would need to specify the height of the horizontal surface relative to the lowest position of the falling block. Fortunately, this is not necessary because the gravitational potential energy associated with the first block does not change.

8.5 Relationship Between Conservative Forces and Potential Energy

In an earlier section we found that the work done on a member of a system by a conservative force between the members does not depend on the path taken by the moving member. The work depends only on the initial and final coordinates. As a consequence, we can define a **potential energy function *U*** such that the work done by a conservative force equals the decrease in the potential energy of the system. Let us imagine a system of particles in which the configuration changes due to the motion of one particle along the x axis. The work done by a conservative force $\mathbf{F}$ as a particle moves along the x axis is[2]

[2] For a general displacement, the work done in two or three dimensions also equals $-\Delta U$, where $U = U(x, y, z)$. We write this formally as $W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = U_i - U_f$.

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\Delta U \tag{8.15}$$

where F_x is the component of **F** in the direction of the displacement. That is, the work done by a conservative force acting between members of a system equals the negative of the change in the potential energy associated with that force when the configuration of the system changes, where the change in the potential energy is defined as $\Delta U = U_f - U_i$. We can also express Equation 8.15 as

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \tag{8.16}$$

Therefore, ΔU is negative when F_x and dx are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

The term *potential energy* implies that the system has the potential, or capability, of either gaining kinetic energy or doing work when it is released under the influence of a conservative force exerted on an object by some other member of the system. It is often convenient to establish some particular location x_i of one member of a system as representing a reference configuration and measure all potential energy differences with respect to it. We can then define the potential energy function as

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i \tag{8.17}$$

The value of U_i is often taken to be zero for the reference configuration. It really does not matter what value we assign to U_i because any nonzero value merely shifts $U_f(x)$ by a constant amount and only the *change* in potential energy is physically meaningful.

If the conservative force is known as a function of position, we can use Equation 8.17 to calculate the change in potential energy of a system as an object within the system moves from x_i to x_f.

If the point of application of the force undergoes an infinitesimal displacement dx, we can express the infinitesimal change in the potential energy of the system dU as

$$dU = -F_x \, dx$$

Therefore, the conservative force is related to the potential energy function through the relationship[3]

$$F_x = -\frac{dU}{dx} \tag{8.18}$$

Relation of force between members of a system to the potential energy of the system

That is, **the *x* component of a conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to *x*.**

We can easily check this relationship for the two examples already discussed. In the case of the deformed spring, $U_s = \frac{1}{2}kx^2$, and therefore

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$

which corresponds to the restoring force in the spring (Hooke's law). Because the gravitational potential energy function is $U_g = mgy$, it follows from Equation 8.18 that $F_g = -mg$ when we differentiate U_g with respect to y instead of x.

We now see that U is an important function because a conservative force can be derived from it. Furthermore, Equation 8.18 should clarify the fact that adding a constant to the potential energy is unimportant because the derivative of a constant is zero.

[3] In three dimensions, the expression is, $\mathbf{F} = -\frac{\partial U}{\partial x}\hat{\mathbf{i}} - \frac{\partial U}{\partial y}\hat{\mathbf{j}} - \frac{\partial U}{\partial z}\hat{\mathbf{k}}$ where $\frac{\partial U}{\partial x}$ etc. are partial derivatives. In the language of vector calculus, **F** equals the negative of the *gradient* of the scalar quantity $U(x, y, z)$.

Quick Quiz 8.11 What does the slope of a graph of $U(x)$ versus x represent? (a) the magnitude of the force on the object (b) the negative of the magnitude of the force on the object (c) the x component of the force on the object (d) the negative of the x component of the force on the object.

8.6 Energy Diagrams and Equilibrium of a System

The motion of a system can often be understood qualitatively through a graph of its potential energy versus the position of a member of the system. Consider the potential energy function for a block–spring system, given by $U_s = \frac{1}{2}kx^2$. This function is plotted versus x in Figure 8.16a. (A common mistake is to think that potential energy on the graph represents height. This is clearly not the case here, where the block is only moving horizontally.) The force F_s exerted by the spring on the block is related to U_s through Equation 8.18:

$$F_s = -\frac{dU_s}{dx} = -kx$$

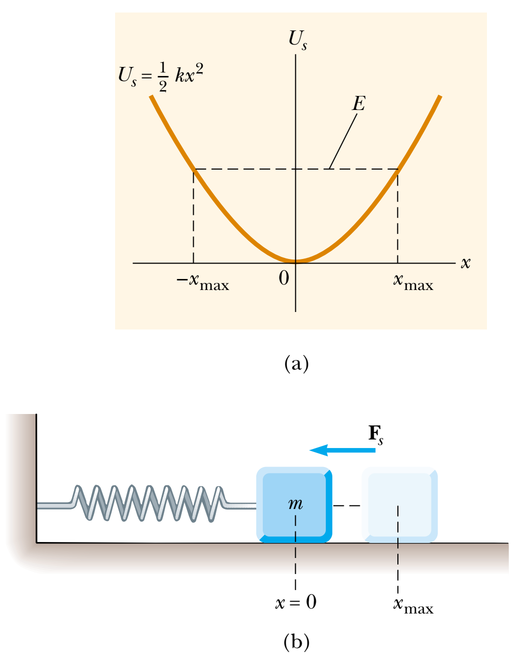
As we saw in Quick Quiz 8.11, the x component of the force is equal to the negative of the slope of the U-versus-x curve. When the block is placed at rest at the equilibrium position of the spring ($x = 0$), where $F_s = 0$, it will remain there unless some external force F_{ext} acts on it. If this external force stretches the spring from equilibrium, x is positive and the slope dU/dx is positive; therefore, the force F_s exerted by the spring is negative and the block accelerates back toward $x = 0$ when released. If the external force compresses the spring, then x is negative and the slope is negative; therefore, F_s is positive and again the mass accelerates toward $x = 0$ upon release.

Stable equilibrium

From this analysis, we conclude that the $x = 0$ position for a block–spring system is one of **stable equilibrium.** That is, any movement away from this position results in a force directed back toward $x = 0$. In general, **configurations of stable equilibrium correspond to those for which $U(x)$ is a minimum.**

From Figure 8.16 we see that if the block is given an initial displacement x_{max} and is released from rest, its total energy initially is the potential energy $\frac{1}{2}kx^2_{max}$ stored in the

At the Active Figures link at http://www.pse6.com, you can observe the block oscillate between its turning points and trace the corresponding points on the potential energy curve for varying values of k.



Active Figure 8.16 (a) Potential energy as a function of x for the frictionless block–spring system shown in (b). The block oscillates between the turning points, which have the coordinates $x = \pm x_{max}$. Note that the restoring force exerted by the spring always acts toward $x = 0$, the position of stable equilibrium.

spring. As the block starts to move, the system acquires kinetic energy and loses an equal amount of potential energy. Because the total energy of the system must remain constant, the block oscillates (moves back and forth) between the two points $x = -x_{max}$ and $x = +x_{max}$, called the *turning points.* In fact, because no energy is lost (no friction), the block will oscillate between $-x_{max}$ and $+x_{max}$ forever. (We discuss these oscillations further in Chapter 15.) From an energy viewpoint, the energy of the system cannot exceed $\frac{1}{2}kx_{max}^2$; therefore, the block must stop at these points and, because of the spring force, must accelerate toward $x = 0$.

Another simple mechanical system that has a configuration of stable equilibrium is a ball rolling about in the bottom of a bowl. Anytime the ball is displaced from its lowest position, it tends to return to that position when released.

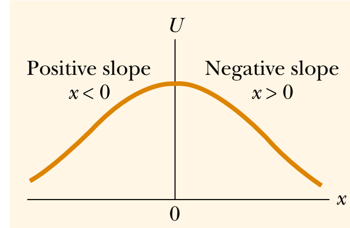


Figure 8.17 A plot of U versus x for a particle that has a position of unstable equilibrium located at $x = 0$. For any finite displacement of the particle, the force on the particle is directed away from $x = 0$.

Now consider a particle moving along the x axis under the influence of a conservative force F_x, where the U-versus-x curve is as shown in Figure 8.17. Once again, $F_x = 0$ at $x = 0$, and so the particle is in equilibrium at this point. However, this is a position of **unstable equilibrium** for the following reason: Suppose that the particle is displaced to the right ($x > 0$). Because the slope is negative for $x > 0$, $F_x = -dU/dx$ is positive, and the particle accelerates away from $x = 0$. If instead the particle is at $x = 0$ and is displaced to the left ($x < 0$), the force is negative because the slope is positive for $x < 0$, and the particle again accelerates away from the equilibrium position. The position $x = 0$ in this situation is one of unstable equilibrium because for any displacement from this point, the force pushes the particle farther away from equilibrium. The force pushes the particle toward a position of lower potential energy. A pencil balanced on its point is in a position of unstable equilibrium. If the pencil is displaced slightly from its absolutely vertical position and is then released, it will surely fall over. In general, **configurations of unstable equilibrium correspond to those for which $U(x)$ is a maximum.**

Unstable equilibrium

Finally, a situation may arise where U is constant over some region. This is called a configuration of **neutral equilibrium.** Small displacements from a position in this region produce neither restoring nor disrupting forces. A ball lying on a flat horizontal surface is an example of an object in neutral equilibrium.

Neutral equilibrium

Example 8.11 Force and Energy on an Atomic Scale

The potential energy associated with the force between two neutral atoms in a molecule can be modeled by the Lennard–Jones potential energy function:

$$U(x) = 4\epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

where x is the separation of the atoms. The function $U(x)$ contains two parameters σ and ϵ that are determined from experiments. Sample values for the interaction between two atoms in a molecule are $\sigma = 0.263$ nm and $\epsilon = 1.51 \times 10^{-22}$ J.

(A) Using a spreadsheet or similar tool, graph this function and find the most likely distance between the two atoms.

Solution We expect to find stable equilibrium when the two atoms are separated by some equilibrium distance and the potential energy of the system of two atoms (the molecule) is a minimum. One can minimize the function $U(x)$ by taking its derivative and setting it equal to zero:

$$\frac{dU(x)}{dx} = 4\epsilon\frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right] = 0$$

$$= 4\epsilon\left[\frac{-12\sigma^{12}}{x^{13}} - \frac{-6\sigma^{6}}{x^{7}}\right] = 0$$

Solving for x—the equilibrium separation of the two atoms in the molecule—and inserting the given information yields

$$x = \boxed{2.95 \times 10^{-10}\ \text{m.}}$$

We graph the Lennard–Jones function on both sides of this critical value to create our energy diagram, as shown in Figure 8.18a. Notice that $U(x)$ is extremely large when the atoms are very close together, is a minimum when the atoms are at their critical separation, and then increases again as the atoms move apart. When $U(x)$ is a minimum, the atoms are in stable equilibrium; this indicates that this is the most likely separation between them.

(B) Determine $F_x(x)$—the force that one atom exerts on the other in the molecule as a function of separation—and argue that the way this force behaves is physically plausible when the atoms are close together and far apart.

Solution Because the atoms combine to form a molecule, the force must be attractive when the atoms are far apart. On the other hand, the force must be repulsive when the two atoms are very close together. Otherwise, the molecule would collapse in on itself. Thus, the force must change sign at the critical separation, similar to the way spring forces switch sign in the change from extension to compression. Applying Equation 8.18 to the Lennard–Jones potential energy function gives

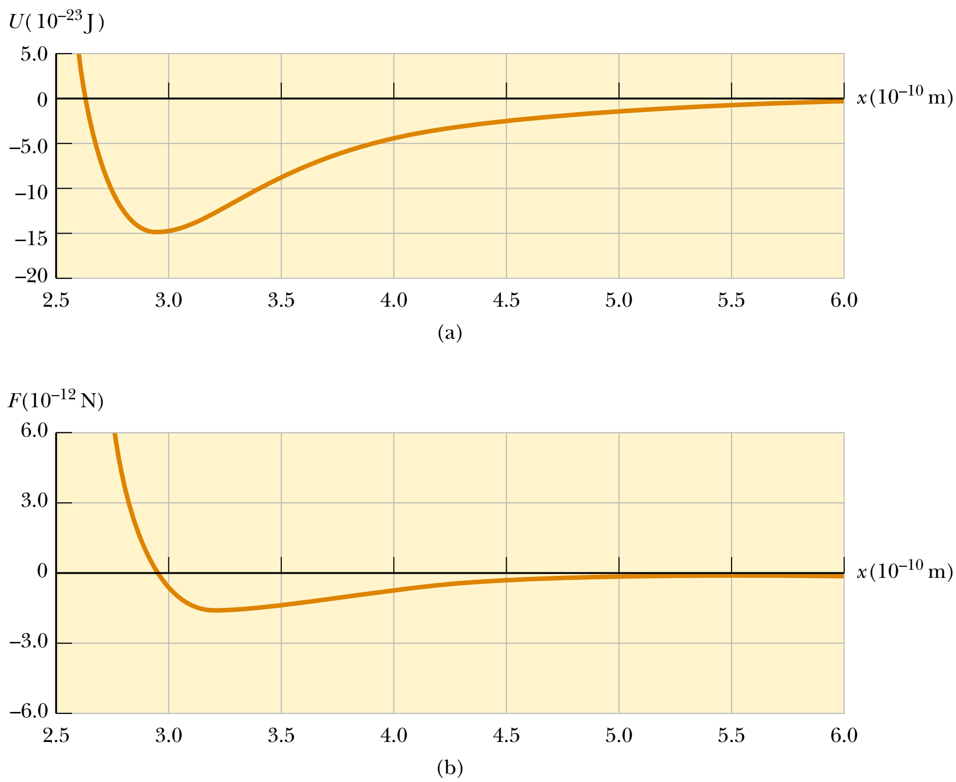


Figure 8.18 (Example 8.11) (a) Potential energy curve associated with a molecule. The distance x is the separation between the two atoms making up the molecule. (b) Force exerted on one atom by the other.

$$F_x = -\frac{dU(x)}{dx} = -4\epsilon \frac{d}{dx}\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

$$= \boxed{4\epsilon\left[\frac{12\sigma^{12}}{x^{13}} - \frac{6\sigma^{6}}{x^{7}}\right]}$$

This result is graphed in Figure 8.18b. As expected, the force is positive (repulsive) at small atomic separations, zero when the atoms are at the position of stable equilibrium [recall how we found the minimum of $U(x)$], and negative (attractive) at greater separations. Note that the force approaches zero as the separation between the atoms becomes very great.

SUMMARY

Take a Practice Test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

If a particle of mass m is at a distance y above the Earth's surface, the **gravitational potential energy** of the particle–Earth system is

$$U_g \equiv mgy \tag{8.2}$$

The **elastic potential energy** stored in a spring of force constant k is

$$U_s \equiv \tfrac{1}{2}kx^2 \tag{8.11}$$

A reference configuration of the system should be chosen, and this configuration is often assigned a potential energy of zero.

A force is **conservative** if the work it does on a particle moving between two points is independent of the path the particle takes between the two points. Furthermore, a force is conservative if the work it does on a particle is zero when the particle moves through an arbitrary closed path and returns to its initial position. A force that does not meet these criteria is said to be **nonconservative.**

The **total mechanical energy of a system** is defined as the sum of the kinetic energy and the potential energy:

$$E_{mech} \equiv K + U \tag{8.8}$$

If a system is isolated and if no nonconservative forces are acting on objects inside the system, then the total mechanical energy of the system is constant:

$$K_f + U_f = K_i + U_i \tag{8.9}$$

If nonconservative forces (such as friction) act on objects inside a system, then mechanical energy is not conserved. In these situations, the difference between the total final mechanical energy and the total initial mechanical energy of the system equals the energy transformed to internal energy by the nonconservative forces.

A **potential energy function** U can be associated only with a conservative force. If a conservative force **F** acts between members of a system while one member moves along the x axis from x_i to x_f, then the change in the potential energy of the system equals the negative of the work done by that force:

$$U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx \tag{8.16}$$

Systems can be in three types of equilibrium configurations when the net force on a member of the system is zero. Configurations of **stable equilibrium** correspond to those for which $U(x)$ is a minimum. Configurations of **unstable equilibrium** correspond to those for which $U(x)$ is a maximum. **Neutral equilibrium** arises where U is constant as a member of the system moves over some region.

QUESTIONS

1. If the height of a playground slide is kept constant, will the length of the slide or the presence of bumps make any difference in the final speed of children playing on it? Assume the slide is slick enough to be considered frictionless. Repeat this question assuming friction is present.
2. Explain why the total energy of a system can be either positive or negative, whereas the kinetic energy is always positive.
3. One person drops a ball from the top of a building while another person at the bottom observes its motion. Will these two people agree on the value of the gravitational potential energy of the ball–Earth system? On the change in potential energy? On the kinetic energy?
4. Discuss the changes in mechanical energy of an object–Earth system in (a) lifting the object, (b) holding the object at a fixed position, and (c) lowering the object slowly. Include the muscles in your discussion.
5. In Chapter 7, the work–kinetic energy theorem, $W = \Delta K$, was introduced. This equation states that work done on a system appears as a change in kinetic energy. This is a special-case equation, valid if there are no changes in any other type of energy such as potential or internal. Give some examples in which work is done on a system, but the change in energy of the system is not that of kinetic energy.
6. If three conservative forces and one nonconservative force act within a system, how many potential-energy terms appear in the equation that describes the system?
7. If only one external force acts on a particle, does it necessarily change the particle's (a) kinetic energy? (b) velocity?
8. A driver brings an automobile to a stop. If the brakes lock so that the car skids, where is the original kinetic energy of the car, and in what form is it after the car stops? Answer the same question for the case in which the brakes do not lock, but the wheels continue to turn.
9. You ride a bicycle. In what sense is your bicycle solar-powered?
10. In an earthquake, a large amount of energy is "released" and spreads outward, potentially causing severe damage. In what form does this energy exist before the earthquake, and by what energy transfer mechanism does it travel?
11. A bowling ball is suspended from the ceiling of a lecture hall by a strong cord. The ball is drawn away from its equilibrium position and released from rest at the tip of the demonstrator's nose as in Figure Q8.11. If the demonstrator remains stationary, explain why she is not struck by the ball on its return swing. Would this demonstrator be safe if the ball were given a push from its starting position at her nose?
12. Roads going up mountains are formed into switchbacks, with the road weaving back and forth along the face of the slope such that there is only a gentle rise on any portion of the roadway. Does this require any less work to be done by an automobile climbing the mountain compared to driving on a roadway that is straight up the slope? Why are switchbacks used?
13. As a sled moves across a flat snow-covered field at constant velocity, is any work done? How does air resistance enter into the picture?
14. You are working in a library, reshelving books. You lift a book from the floor to the top shelf. The kinetic energy of the book on the floor was zero, and the kinetic energy of the book on the top shelf is zero, so there is no change

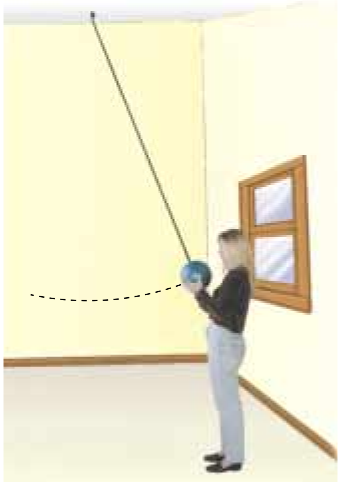


Figure Q8.11

in kinetic energy. Yet you did some work in lifting the book. Is the work–kinetic energy theorem violated?

15. A ball is thrown straight up into the air. At what position is its kinetic energy a maximum? At what position is the gravitational potential energy of the ball–Earth system a maximum?

16. A pile driver is a device used to drive objects into the Earth by repeatedly dropping a heavy weight on them. By how much does the energy of the pile driver–Earth system increase when the weight it drops is doubled? Assume the weight is dropped from the same height each time.

17. Our body muscles exert forces when we lift, push, run, jump, and so forth. Are these forces conservative?

18. A block is connected to a spring that is suspended from the ceiling. If the block is set in motion and air resistance is neglected, describe the energy transformations that occur within the system consisting of the block, Earth, and spring.

19. Describe the energy transformations that occur during (a) the pole vault (b) the shot put (c) the high jump. What is the source of energy in each case?

20. Discuss the energy transformations that occur during the operation of an automobile.

21. What would the curve of U versus x look like if a particle were in a region of neutral equilibrium?

22. A ball rolls on a horizontal surface. Is the ball in stable, unstable, or neutral equilibrium?

23. Consider a ball fixed to one end of a rigid rod whose other end pivots on a horizontal axis so that the rod can rotate in a vertical plane. What are the positions of stable and unstable equilibrium?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging □ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 8.1 Potential Energy of a System

1. A 1 000-kg roller coaster train is initially at the top of a rise, at point Ⓐ. It then moves 135 ft, at an angle of 40.0° below the horizontal, to a lower point Ⓑ. (a) Choose point Ⓑ to be the zero level for gravitational potential energy. Find the potential energy of the roller coaster–Earth system at points Ⓐ and Ⓑ, and the change in potential energy as the coaster moves. (b) Repeat part (a), setting the zero reference level at point Ⓐ.

2. A 400-N child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.

3. A person with a remote mountain cabin plans to install her own hydroelectric plant. A nearby stream is 3.00 m wide and 0.500 m deep. Water flows at 1.20 m/s over the brink of a waterfall 5.00 m high. The manufacturer promises only 25.0% efficiency in converting the potential energy of the water–Earth system into electric energy. Find the power she can generate. (Large-scale hydroelectric plants, with a much larger drop, are more efficient.)

Section 8.2 The Isolated System–Conservation of Mechanical Energy

4. At 11:00 A.M. on September 7, 2001, more than 1 million British school children jumped up and down for one minute. The curriculum focus of the "Giant Jump" was on earthquakes, but it was integrated with many other topics, such as exercise, geography, cooperation, testing hypotheses, and setting world records. Children built their own seismographs, which registered local effects. (a) Find the mechanical energy released in the experiment. Assume that 1 050 000 children of average mass 36.0 kg jump twelve times each, raising their centers of mass by 25.0 cm each time and briefly resting between one jump and the next. The free-fall acceleration in Britain is 9.81 m/s^2. (b) Most of the energy is converted very rapidly into internal energy within the bodies of the children and the floors of the school buildings. Of the energy that propagates into the ground, most produces high-frequency "microtremor" vibrations that are rapidly damped and cannot travel far. Assume that 0.01% of the energy is carried away by a long-range seismic wave. The magnitude of an earthquake on the Richter scale is given by

$$M = \frac{\log E - 4.8}{1.5}$$

where E is the seismic wave energy in joules. According to this model, what is the magnitude of the demonstration quake? (It did not register above background noise overseas or on the seismograph of the Wolverton Seismic Vault, Hampshire.)

5. A bead slides without friction around a loop-the-loop (Fig. P8.5). The bead is released from a height $h = 3.50R$. (a) What is its speed at point Ⓐ? (b) How large is the normal force on it if its mass is 5.00 g?

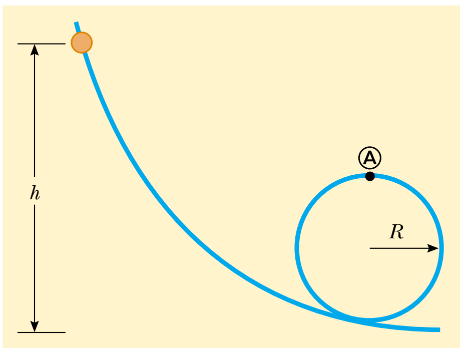


Figure P8.5

6. Dave Johnson, the bronze medalist at the 1992 Olympic decathlon in Barcelona, leaves the ground at the high jump with vertical velocity component 6.00 m/s. How far does his center of mass move up as he makes the jump?

7. A glider of mass 0.150 kg moves on a horizontal frictionless air track. It is permanently attached to one end of a massless horizontal spring, which has a force constant of 10.0 N/m both for extension and for compression. The other end of the spring is fixed. The glider is moved to compress the spring by 0.180 m and then released from rest. Calculate the speed of the glider (a) at the point where it has moved 0.180 m from its starting point, so that the spring is momentarily exerting no force and (b) at the point where it has moved 0.250 m from its starting point.

8. A loaded ore car has a mass of 950 kg and rolls on rails with negligible friction. It starts from rest and is pulled up a mine shaft by a cable connected to a winch. The shaft is inclined at 30.0° above the horizontal. The car accelerates uniformly to a speed of 2.20 m/s in 12.0 s and then continues at constant speed. (a) What power must the winch motor provide when the car is moving at constant speed? (b) What maximum power must the winch motor provide? (c) What total energy transfers out of the motor by work by the time the car moves off the end of the track, which is of length 1 250 m?

9. A simple pendulum, which we will consider in detail in Chapter 15, consists of an object suspended by a string. The object is assumed to be a particle. The string, with its top end fixed, has negligible mass and does not stretch. In the absence of air friction, the system oscillates by swinging back and forth in a vertical plane. If the string is 2.00 m long and makes an initial angle of 30.0° with the vertical, calculate the speed of the particle (a) at the lowest point in its trajectory and (b) when the angle is 15.0°.

10. An object of mass m starts from rest and slides a distance d down a frictionless incline of angle θ. While sliding, it contacts an unstressed spring of negligible mass as shown in Figure P8.10. The object slides an additional distance x as it is brought momentarily to rest by compression of the spring (of force constant k). Find the initial separation d between object and spring.

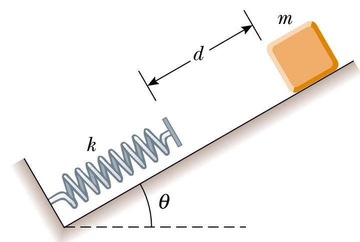


Figure P8.10

11. A block of mass 0.250 kg is placed on top of a light vertical spring of force constant 5 000 N/m and pushed downward so that the spring is compressed by 0.100 m. After the block is released from rest, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise?

12. A circus trapeze consists of a bar suspended by two parallel ropes, each of length ℓ, allowing performers to swing in a vertical circular arc (Figure P8.12). Suppose a performer with mass m holds the bar and steps off an elevated platform, starting from rest with the ropes at an angle θ_i with respect to the vertical. Suppose the size of the performer's body is small compared to the length ℓ, that she does not pump the trapeze to swing higher, and that air resistance is negligible. (a) Show that when the ropes make an angle θ with the vertical, the performer must exert a force

$$mg(3\cos\theta - 2\cos\theta_i)$$

in order to hang on. (b) Determine the angle θ_i for which

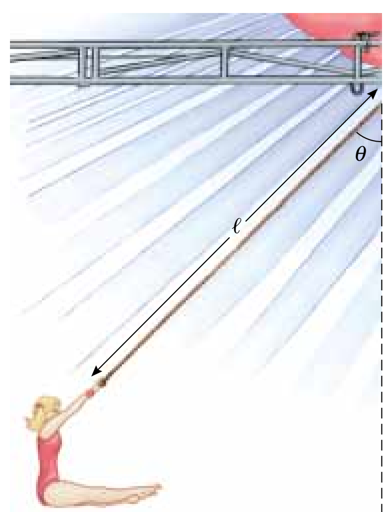


Figure P8.12

the force needed to hang on at the bottom of the swing is twice the performer's weight.

13. Two objects are connected by a light string passing over a light frictionless pulley as shown in Figure P8.13. The object of mass 5.00 kg is released from rest. Using the principle of conservation of energy, (a) determine the speed of the 3.00-kg object just as the 5.00-kg object hits the ground. (b) Find the maximum height to which the 3.00-kg object rises.

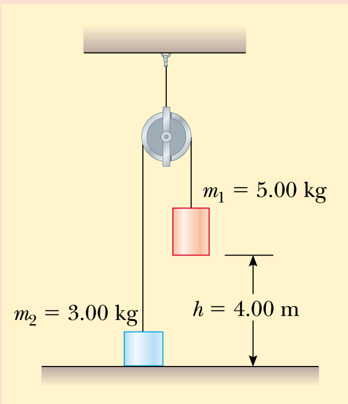


Figure P8.13 Problems 13 and 14.

14. Two objects are connected by a light string passing over a light frictionless pulley as in Figure P8.13. The object of mass m_1 is released from rest at height h. Using the principle of conservation of energy, (a) determine the speed of m_2 just as m_1 hits the ground. (b) Find the maximum height to which m_2 rises.

15. A light rigid rod is 77.0 cm long. Its top end is pivoted on a low-friction horizontal axle. The rod hangs straight down at rest with a small massive ball attached to its bottom end. You strike the ball, suddenly giving it a horizontal velocity so that it swings around in a full circle. What minimum speed at the bottom is required to make the ball go over the top of the circle?

16. Air moving at 11.0 m/s in a steady wind encounters a windmill of diameter 2.30 m and having an efficiency of 27.5%. The energy generated by the windmill is used to pump water from a well 35.0 m deep into a tank 2.30 m above the ground. At what rate in liters per minute can water be pumped into the tank?

17. A 20.0-kg cannon ball is fired from a cannon with muzzle speed of 1 000 m/s at an angle of 37.0° with the horizontal. A second ball is fired at an angle of 90.0°. Use the conservation of energy principle to find (a) the maximum height reached by each ball and (b) the total mechanical energy at the maximum height for each ball. Let $y = 0$ at the cannon.

18. A 2.00-kg ball is attached to the bottom end of a length of fishline with a breaking strength of 10 lb (44.5 N). The top end of the fishline is held stationary. The ball is released from rest with the line taut and horizontal ($\theta = 90.0°$). At what angle θ (measured from the vertical) will the fishline break?

19. A daredevil plans to bungee-jump from a balloon 65.0 m above a carnival midway (Figure P8.19). He will use a uniform elastic cord, tied to a harness around his body, to stop his fall at a point 10.0 m above the ground. Model his body as a particle and the cord as having negligible mass and obeying Hooke's force law. In a preliminary test, hanging at rest from a 5.00-m length of the cord, he finds that his body weight stretches it by 1.50 m. He will drop from rest at the point where the top end of a longer section of the cord is attached to the stationary balloon. (a) What length of cord should he use? (b) What maximum acceleration will he experience?

Gamma

Figure P8.19

20. **Review problem.** The system shown in Figure P8.20 consists of a light inextensible cord, light frictionless pulleys, and blocks of equal mass. It is initially held at rest so that the blocks are at the same height above the ground. The blocks are then released. Find the speed of block A at the moment when the vertical separation of the blocks is h.

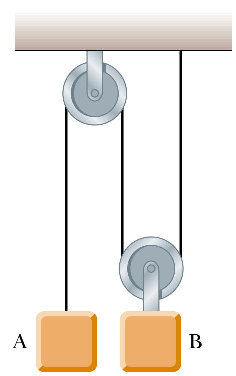


Figure P8.20

Section 8.3 Conservative and Nonconservative Forces

21. A 4.00-kg particle moves from the origin to position C, having coordinates $x = 5.00$ m and $y = 5.00$ m. One force on the particle is the gravitational force acting in the negative y direction (Fig. P8.21). Using Equation 7.3, calculate the

work done by the gravitational force in going from O to C along (a) OAC. (b) OBC. (c) OC. Your results should all be identical. Why?

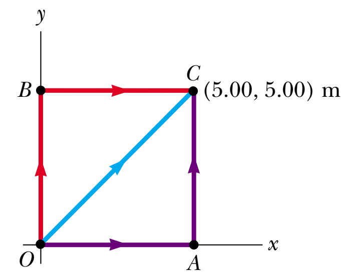


Figure P8.21 Problems 21, 22 and 23.

22. (a) Suppose that a constant force acts on an object. The force does not vary with time, nor with the position or the velocity of the object. Start with the general definition for work done by a force

$$W = \int_i^f \mathbf{F} \cdot d\mathbf{r}$$

and show that the force is conservative. (b) As a special case, suppose that the force $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ N acts on a particle that moves from O to C in Figure P8.21. Calculate the work done by $\mathbf{F}$ if the particle moves along each one of the three paths OAC, OBC, and OC. (Your three answers should be identical.)

23. A force acting on a particle moving in the xy plane is given by $\mathbf{F} = (2y\,\hat{\mathbf{i}} + x^2\,\hat{\mathbf{j}})$ N, where x and y are in meters. The particle moves from the origin to a final position having coordinates $x = 5.00$ m and $y = 5.00$ m, as in Figure P8.21. Calculate the work done by $\mathbf{F}$ along (a) OAC, (b) OBC, (c) OC. (d) Is $\mathbf{F}$ conservative or nonconservative? Explain.

24. A particle of mass $m = 5.00$ kg is released from point Ⓐ and slides on the frictionless track shown in Figure P8.24. Determine (a) the particle's speed at points Ⓑ and Ⓒ and (b) the net work done by the gravitational force in moving the particle from Ⓐ to Ⓒ.

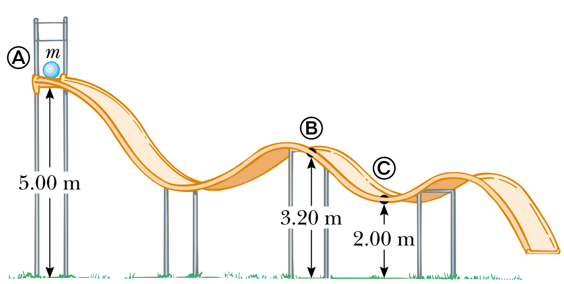


Figure P8.24

25. A single constant force $\mathbf{F} = (3\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ N acts on a 4.00-kg particle. (a) Calculate the work done by this force if the particle moves from the origin to the point having the vector position $\mathbf{r} = (2\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}})$ m. Does this result depend on the path? Explain. (b) What is the speed of the particle at $\mathbf{r}$ if its speed at the origin is 4.00 m/s? (c) What is the change in the potential energy?

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

26. At time t_i, the kinetic energy of a particle is 30.0 J and the potential energy of the system to which it belongs is 10.0 J. At some later time t_f, the kinetic energy of the particle is 18.0 J. (a) If only conservative forces act on the particle, what are the potential energy and the total energy at time t_f? (b) If the potential energy of the system at time t_f is 5.00 J, are there any nonconservative forces acting on the particle? Explain.

27. In her hand a softball pitcher swings a ball of mass 0.250 kg around a vertical circular path of radius 60.0 cm before releasing it from her hand. The pitcher maintains a component of force on the ball of constant magnitude 30.0 N in the direction of motion around the complete path. The speed of the ball at the top of the circle is 15.0 m/s. If she releases the ball at the bottom of the circle, what is its speed upon release?

28. An electric scooter has a battery capable of supplying 120 Wh of energy. If friction forces and other losses account for 60.0% of the energy usage, what altitude change can a rider achieve when driving in hilly terrain, if the rider and scooter have a combined weight of 890 N?

29. The world's biggest locomotive is the MK5000C, a behemoth of mass 160 metric tons driven by the most powerful engine ever used for rail transportation, a Caterpillar diesel capable of 5 000 hp. Such a huge machine can provide a gain in efficiency, but its large mass presents challenges as well. The engineer finds that the locomotive handles differently from conventional units, notably in braking and climbing hills. Consider the locomotive pulling no train, but traveling at 27.0 m/s on a level track while operating with output power 1 000 hp. It comes to a 5.00% grade (a slope that rises 5.00 m for every 100 m along the track). If the throttle is not advanced, so that the power level is held steady, to what value will the speed drop? Assume that friction forces do not depend on the speed.

30. A 70.0-kg diver steps off a 10.0-m tower and drops straight down into the water. If he comes to rest 5.00 m beneath the surface of the water, determine the average resistance force exerted by the water on the diver.

31. The coefficient of friction between the 3.00-kg block and the surface in Figure P8.31 is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?

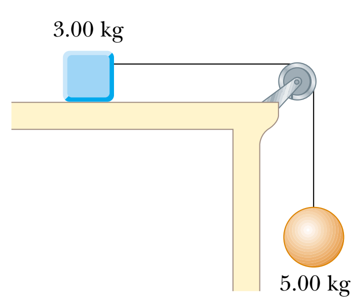


Figure P8.31

32. A boy in a wheelchair (total mass 47.0 kg) wins a race with a skateboarder. The boy has speed 1.40 m/s at the crest of a slope 2.60 m high and 12.4 m long. At the bottom of the slope his speed is 6.20 m/s. If air resistance and rolling resistance can be modeled as a constant friction force of 41.0 N, find the work he did in pushing forward on his wheels during the downhill ride.

33. A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s (Fig. P8.33). The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine (a) the change in the block's kinetic energy, (b) the change in the potential energy of the block–Earth system, and (c) the friction force exerted on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

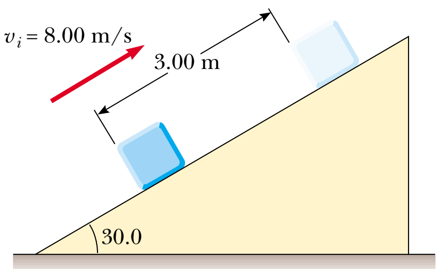


Figure P8.33

34. An 80.0-kg skydiver jumps out of a balloon at an altitude of 1 000 m and opens the parachute at an altitude of 200 m. (a) Assuming that the total retarding force on the diver is constant at 50.0 N with the parachute closed and constant at 3 600 N with the parachute open, what is the speed of the diver when he lands on the ground? (b) Do you think the skydiver will be injured? Explain. (c) At what height should the parachute be opened so that the final speed of the skydiver when he hits the ground is 5.00 m/s? (d) How realistic is the assumption that the total retarding force is constant? Explain.

35. A toy cannon uses a spring to project a 5.30-g soft rubber ball. The spring is originally compressed by 5.00 cm and has a force constant of 8.00 N/m. When the cannon is fired, the ball moves 15.0 cm through the horizontal barrel of the cannon, and there is a constant friction force of 0.032 0 N between the barrel and the ball. (a) With what speed does the projectile leave the barrel of the cannon? (b) At what point does the ball have maximum speed? (c) What is this maximum speed?

36. A 50.0-kg block and a 100-kg block are connected by a string as in Figure P8.36. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50.0 kg block and incline is 0.250. Determine the change in the kinetic energy of the 50.0-kg block as it moves from Ⓐ to Ⓑ, a distance of 20.0 m.

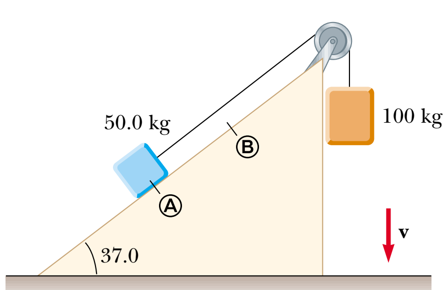


Figure P8.36

37. A 1.50-kg object is held 1.20 m above a relaxed massless vertical spring with a force constant of 320 N/m. The object is dropped onto the spring. (a) How far does it compress the spring? (b) **What If?** How far does it compress the spring if the same experiment is performed on the Moon, where $g = 1.63\ \text{m/s}^2$? (c) **What If?** Repeat part (a), but this time assume a constant air-resistance force of 0.700 N acts on the object during its motion.

38. A 75.0-kg skysurfer is falling straight down with terminal speed 60.0 m/s. Determine the rate at which the skysurfer–Earth system is losing mechanical energy.

39. A uniform board of length L is sliding along a smooth (frictionless) horizontal plane as in Figure P8.39a. The board then slides across the boundary with a rough horizontal surface. The coefficient of kinetic friction between the board and the second surface is μ_k. (a) Find the acceleration of the board at the moment its front end has traveled a distance x beyond the boundary. (b) The board stops at the moment its back end reaches the boundary, as in Figure P8.39b. Find the initial speed v of the board.

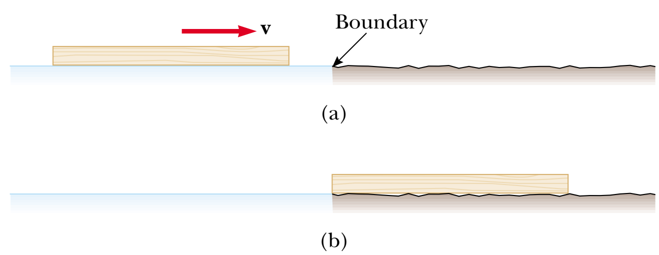


Figure P8.39

Section 8.5 Relationship Between Conservative Forces and Potential Energy

40. A single conservative force acting on a particle varies as $\mathbf{F} = (-Ax + Bx^2)\hat{\mathbf{i}}$ N, where A and B are constants and x is in meters. (a) Calculate the potential-energy function $U(x)$ associated with this force, taking $U = 0$ at $x = 0$. (b) Find the change in potential energy and the change in kinetic energy as the particle moves from $x = 2.00$ m to $x = 3.00$ m.

41. A single conservative force acts on a 5.00-kg particle. The equation $F_x = (2x + 4)$ N describes the force, where x is in meters. As the particle moves along the x axis from $x = 1.00$ m to $x = 5.00$ m, calculate (a) the work done by this force, (b) the change in the potential energy of the system, and (c) the kinetic energy of the particle at $x = 5.00$ m if its speed is 3.00 m/s at $x = 1.00$ m.

42. A potential-energy function for a two-dimensional force is of the form $U = 3x^3y - 7x$. Find the force that acts at the point (x, y).

43. The potential energy of a system of two particles separated by a distance r is given by $U(r) = A/r$, where A is a constant. Find the radial force $\mathbf{F}_r$ that each particle exerts on the other.

Section 8.6 Energy Diagrams and Equilibrium of a System

44. A right circular cone can be balanced on a horizontal surface in three different ways. Sketch these three equilibrium configurations, and identify them as positions of stable, unstable, or neutral equilibrium.

45. For the potential energy curve shown in Figure P8.45, (a) determine whether the force F_x is positive, negative, or zero at the five points indicated. (b) Indicate points of stable, unstable, and neutral equilibrium. (c) Sketch the curve for F_x versus x from $x = 0$ to $x = 9.5$ m.

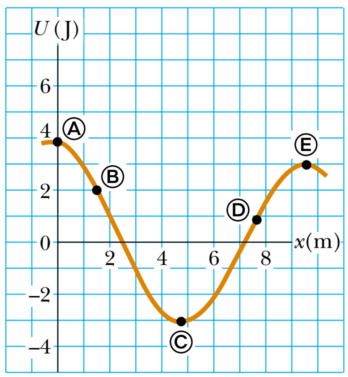


Figure P8.45

46. A particle moves along a line where the potential energy of its system depends on its position r as graphed in Figure P8.46. In the limit as r increases without bound, $U(r)$ approaches +1 J. (a) Identify each equilibrium position for this particle. Indicate whether each is a point of stable, unstable, or neutral equilibrium. (b) The particle will be bound if the total energy of the system is in what range? Now suppose that the system has energy −3 J. Determine (c) the range of positions where the particle can be found, (d) its maximum kinetic energy, (e) the location where it has maximum kinetic energy, and (f) the *binding energy* of the system—that is, the additional energy that it would have to be given in order for the particle to move out to $r \to \infty$.

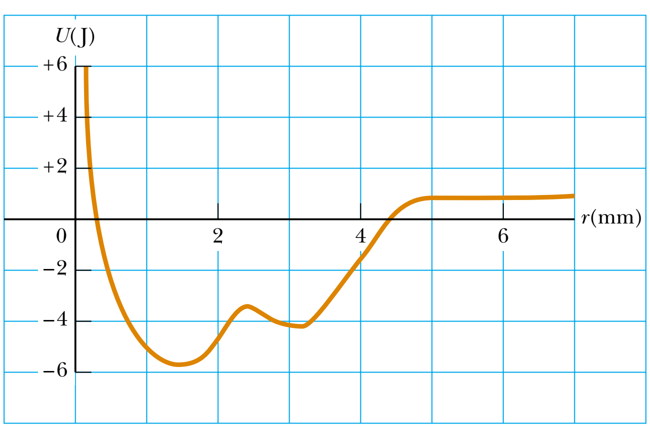
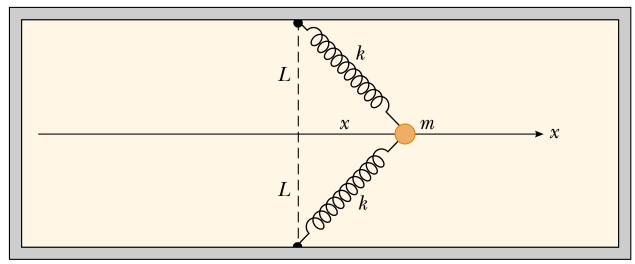


Figure P8.46

47. A particle of mass 1.18 kg is attached between two identical springs on a horizontal frictionless tabletop. The springs have force constant k and each is initially unstressed. (a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as in Figure P8.47, show that the potential energy of the system is

$$U(x) = kx^2 + 2kL\left(L - \sqrt{x^2 + L^2}\right)$$

(*Hint*: See Problem 58 in Chapter 7.) (b) Make a plot of $U(x)$ versus x and identify all equilibrium points. Assume that $L = 1.20$ m and $k = 40.0$ N/m. (c) If the particle is pulled 0.500 m to the right and then released, what is its speed when it reaches the equilibrium point $x = 0$?



Top View

Figure P8.47

Additional Problems

48. A block slides down a curved frictionless track and then up an inclined plane as in Figure P8.48. The coefficient of kinetic friction between block and incline is μ_k. Use energy methods to show that the maximum height reached by the block is

$$y_{\max} = \frac{h}{1 + \mu_k \cot \theta}$$

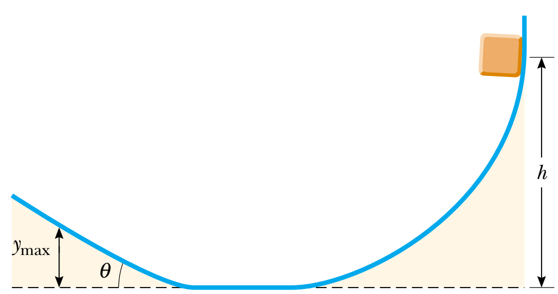


Figure P8.48

49. Make an order-of-magnitude estimate of your power output as you climb stairs. In your solution, state the physical quantities you take as data and the values you measure or estimate for them. Do you consider your peak power or your sustainable power?

50. **Review problem.** The mass of a car is 1 500 kg. The shape of the body is such that its aerodynamic drag coefficient is $D = 0.330$ and the frontal area is 2.50 m^2. Assuming that the drag force is proportional to v^2 and neglecting other sources of friction, calculate the power required to maintain a speed of 100 km/h as the car climbs a long hill sloping at 3.20°.

51. Assume that you attend a state university that started out as an agricultural college. Close to the center of the campus is a tall silo topped with a hemispherical cap. The cap is frictionless when wet. Someone has somehow balanced a pumpkin at the highest point. The line from the center of curvature of the cap to the pumpkin makes an angle $\theta_i = 0°$ with the vertical. While you happen to be standing nearby in the middle of a rainy night, a breath of wind makes the pumpkin start sliding downward from rest. It loses contact with the cap when the line from the center of the hemisphere to the pumpkin makes a certain angle with the vertical. What is this angle?

52. A 200-g particle is released from rest at point Ⓐ along the horizontal diameter on the inside of a frictionless, hemispherical bowl of radius $R = 30.0$ cm (Fig. P8.52). Calculate (a) the gravitational potential energy of the particle–Earth system when the particle is at point Ⓐ relative to point Ⓑ, (b) the kinetic energy of the particle at point Ⓑ, (c) its speed at point Ⓑ, and (d) its kinetic energy and the potential energy when the particle is at point Ⓒ.

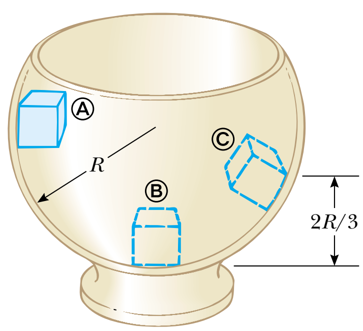


Figure P8.52 Problems 52 and 53.

53. **What If?** The particle described in Problem 52 (Fig. P8.52) is released from rest at Ⓐ, and the surface of the bowl is rough. The speed of the particle at Ⓑ is 1.50 m/s. (a) What is its kinetic energy at Ⓑ? (b) How much mechanical energy is transformed into internal energy as the particle moves from Ⓐ to Ⓑ? (c) Is it possible to determine the coefficient of friction from these results in any simple manner? Explain.

54. A 2.00-kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N/m (Fig. P8.54). The pulley is frictionless. The block is released from rest when the spring is unstretched. The block moves 20.0 cm down the incline before coming to rest. Find the coefficient of kinetic friction between block and incline.

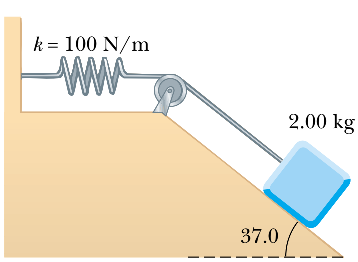


Figure P8.54 Problems 54 and 55.

55. **Review problem.** Suppose the incline is frictionless for the system described in Problem 54 (Fig. P8.54). The block is released from rest with the spring initially unstretched. (a) How far does it move down the incline before coming to rest? (b) What is its acceleration at its lowest point? Is the acceleration constant? (c) Describe the energy transformations that occur during the descent.

56. A child's pogo stick (Fig. P8.56) stores energy in a spring with a force constant of 2.50×10^4 N/m. At position Ⓐ ($x_A = -0.100$ m), the spring compression is a maximum and the child is momentarily at rest. At position Ⓑ ($x_B = 0$), the spring is relaxed and the child is moving upward. At position Ⓒ, the child is again momentarily at rest at the top of the jump. The combined mass of child and pogo stick is 25.0 kg. (a) Calculate the total energy of the child–stick–Earth system if both gravitational and elastic potential energies are zero for $x = 0$. (b) Determine x_C. (c) Calculate the speed of the child at $x = 0$. (d) Determine the value of x for which the kinetic energy of the system is a maximum. (e) Calculate the child's maximum upward speed.

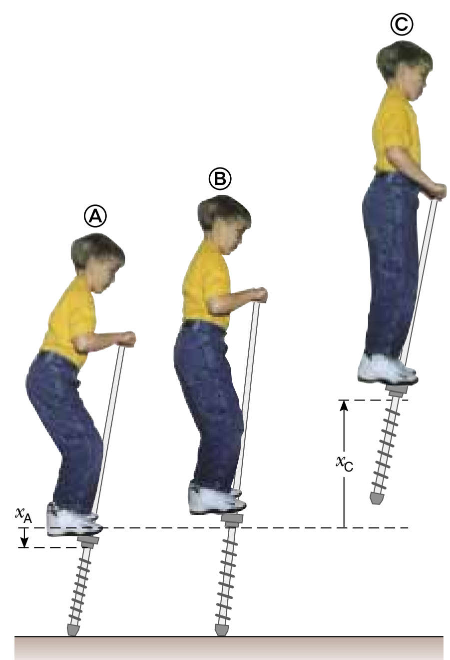


Figure P8.56

57. A 10.0-kg block is released from point Ⓐ in Figure P8.57. The track is frictionless except for the portion between points Ⓑ and Ⓒ, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2 250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between Ⓑ and Ⓒ.

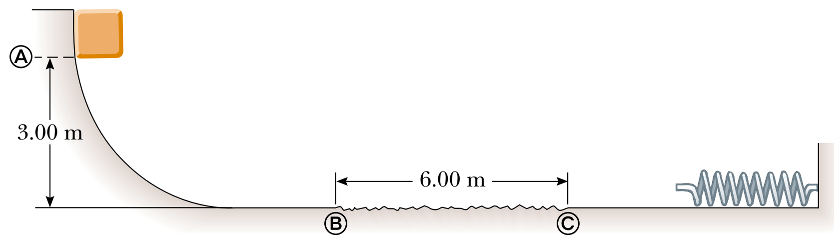


Figure P8.57

58. The potential energy function for a system is given by $U(x) = -x^3 + 2x^2 + 3x$. (a) Determine the force F_x as a function of x. (b) For what values of x is the force equal to zero? (c) Plot $U(x)$ versus x and F_x versus x, and indicate points of stable and unstable equilibrium.

59. A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a light frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m, as shown in Figure P8.59. The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and released from rest. Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).

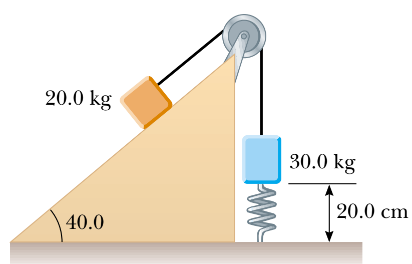


Figure P8.59

60. A 1.00-kg object slides to the right on a surface having a coefficient of kinetic friction 0.250 (Fig. P8.60). The object has a speed of $v_i = 3.00$ m/s when it makes contact with a light spring that has a force constant of 50.0 N/m. The object comes to rest after the spring has been compressed a distance d. The object is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. Finally, the object comes to rest a distance D to the left of the unstretched spring. Find (a) the distance of compression d, (b) the speed v at the unstretched position when the object is moving to the left, and (c) the distance D where the object comes to rest.

61. A block of mass 0.500 kg is pushed against a horizontal spring of negligible mass until the spring is compressed a distance x (Fig. P8.61). The force constant of the spring is 450 N/m. When it is released, the block travels along a frictionless, horizontal surface to point B, the bottom of a vertical circular track of radius $R = 1.00$ m, and continues to move up the track. The speed of the block at the bottom of the track is $v_B = 12.0$ m/s, and the block experiences an average friction force of 7.00 N while sliding up the track. (a) What is x? (b) What speed do you predict for the block at the top of the track? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top?

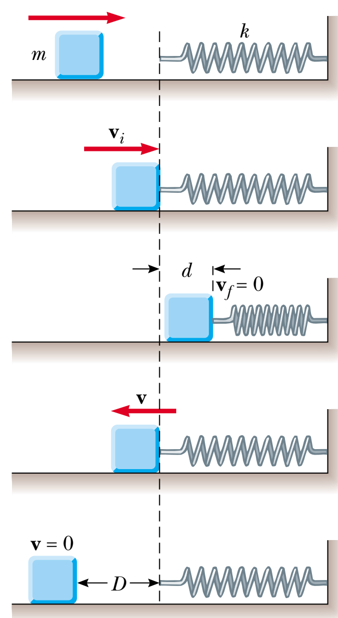


Figure P8.60

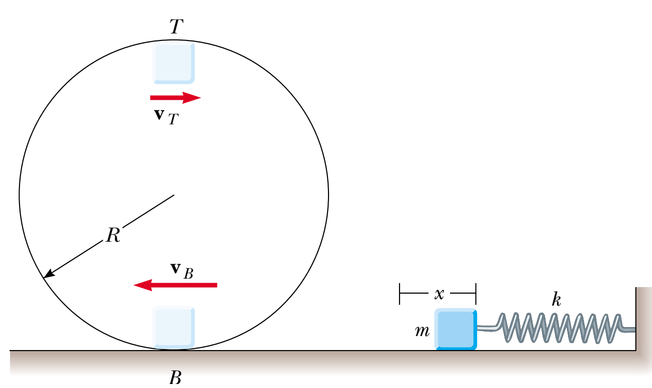


Figure P8.61

62. A uniform chain of length 8.00 m initially lies stretched out on a horizontal table. (a) If the coefficient of static friction between chain and table is 0.600, show that the chain will begin to slide off the table if at least 3.00 m of it hangs over the edge of the table. (b) Determine the speed of the chain as all of it leaves the table, given that the coefficient of kinetic friction between the chain and the table is 0.400.

63. A child slides without friction from a height h along a curved water slide (Fig. P8.63). She is launched from a height $h/5$ into the pool. Determine her maximum airborne height y in terms of h and θ.

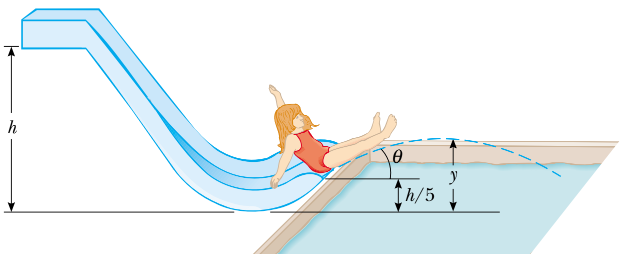


Figure P8.63

64. Refer to the situation described in Chapter 5, Problem 65. A 1.00-kg glider on a horizontal air track is pulled by a string at angle θ. The taut string runs over a light pulley at height h_0 = 40.0 cm above the line of motion of the glider. The other end of the string is attached to a hanging mass of 0.500 kg as in Fig. P5.65. (a) Show that the speed of the glider v_x and the speed of the hanging mass v_y are related by $v_y = v_x \cos\theta$. The glider is released from rest when $\theta = 30.0°$. Find (b) v_x and (c) v_y when $\theta = 45.0°$. (d) Explain why the answers to parts (b) and (c) to Chapter 5, Problem 65 do not help to solve parts (b) and (c) of this problem.

65. Jane, whose mass is 50.0 kg, needs to swing across a river (having width D) filled with man-eating crocodiles to save Tarzan from danger. She must swing into a wind exerting constant horizontal force $\mathbf{F}$, on a vine having length L and initially making an angle θ with the vertical (Fig. P8.65). Taking D = 50.0 m, F = 110 N, L = 40.0 m, and $\theta = 50.0°$, (a) with what minimum speed must Jane begin her swing in order to just make it to the other side? (b) Once the rescue is complete, Tarzan and Jane must swing back across the river. With what minimum speed must they begin their swing? Assume that Tarzan has a mass of 80.0 kg.

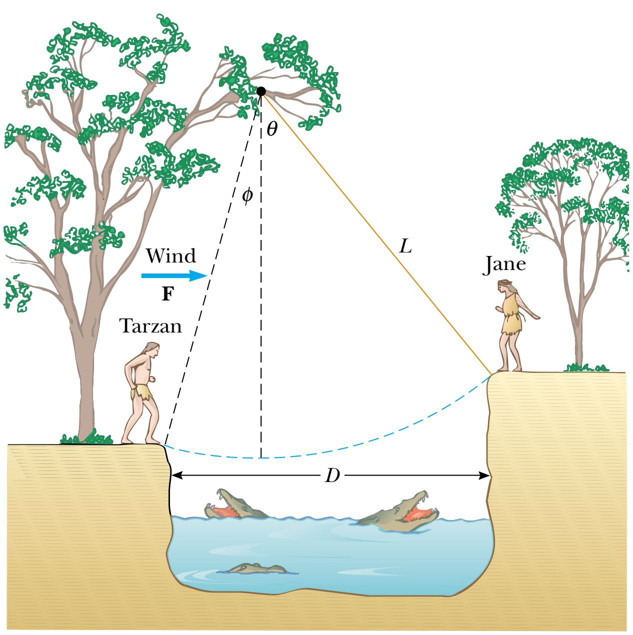


Figure P8.65

66. A 5.00-kg block free to move on a horizontal, frictionless surface is attached to one end of a light horizontal spring. The other end of the spring is held fixed. The spring is compressed 0.100 m from equilibrium and released. The speed of the block is 1.20 m/s when it passes the equilibrium position of the spring. The same experiment is now repeated with the frictionless surface replaced by a surface for which the coefficient of kinetic friction is 0.300. Determine the speed of the block at the equilibrium position of the spring.

67. A skateboarder with his board can be modeled as a particle of mass 76.0 kg, located at his center of mass (which we will study in Chapter 9). As in Figure P8.67, the skateboarder starts from rest in a crouching position at one lip of a half-pipe (point Ⓐ). The half-pipe is a dry water channel, forming one half of a cylinder of radius 6.80 m with its axis horizontal. On his descent, the skateboarder moves without friction so that his center of mass moves through one quarter of a circle of radius 6.30 m. (a) Find his speed at the bottom of the half-pipe (point Ⓑ). (b) Find his centripetal acceleration. (c) Find the normal force n_B acting on the skateboarder at point Ⓑ. Immediately after passing point Ⓑ, he stands up and raises his arms, lifting his center of mass from 0.500 m to 0.950 m above the concrete (point Ⓒ). To account for the conversion of chemical into mechanical energy, model his legs as doing work by pushing him vertically up, with a constant force equal to the normal force n_B, over a distance of 0.450 m. (You will be able to solve this problem with a more accurate model in Chapter 11.) (d) What is the work done on the skateboarder's body in this process? Next, the skateboarder glides upward with his center of mass moving in a quarter circle of radius 5.85 m. His body is horizontal when he passes point Ⓓ, the far lip of the half-pipe. (e) Find his speed at this location. At last he goes ballistic, twisting around while his center of mass moves vertically. (f) How high above point Ⓓ does he rise? (g) Over what time interval is he airborne before he touches down, 2.34 m below the level of point Ⓓ? [*Caution*: Do not try this yourself without the required skill and protective equipment, or in a drainage channel to which you do not have legal access.]

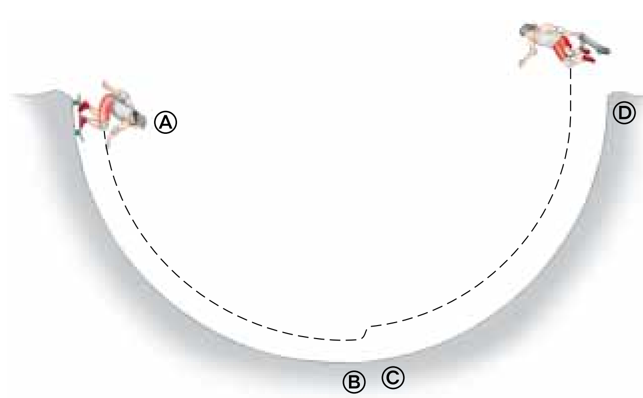


Figure P8.67

68. A block of mass M rests on a table. It is fastened to the lower end of a light vertical spring. The upper end of the spring is fastened to a block of mass m. The upper block is pushed down by an additional force $3mg$, so the spring compression is $4mg/k$. In this configuration the upper block is released from rest. The spring lifts the lower block off the table. In terms of m, what is the greatest possible value for M?

69. A ball having mass m is connected by a strong string of length L to a pivot point and held in place in a vertical position. A wind exerting constant force of magnitude F is blowing from left to right as in Figure P8.69a. (a) If the ball is released from rest, show that the maximum height H reached by the ball, as measured from its initial height, is

$$H = \frac{2L}{1 + (mg/F)^2}$$

Check that the above result is valid both for cases when $0 \le H \le L$ and for $L \le H \le 2L$. (b) Compute the value of H using the values $m = 2.00$ kg, $L = 2.00$ m, and $F = 14.7$ N. (c) Using these same values, determine the *equilibrium* height of the ball. (d) Could the equilibrium height ever be larger than L? Explain.

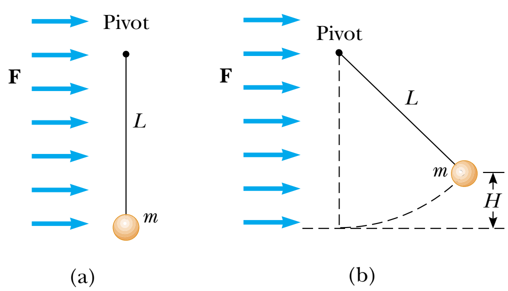


Figure P8.69

70. A ball is tied to one end of a string. The other end of the string is held fixed. The ball is set moving around a vertical circle without friction, and with speed $v_i = \sqrt{Rg}$ at the top of the circle, as in Figure P8.70. At what angle θ should the string be cut so that the ball will then travel through the center of the circle?

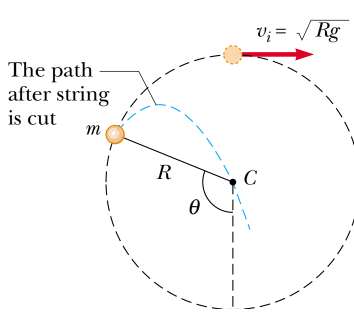


Figure P8.70

71. A ball whirls around in a vertical circle at the end of a string. If the total energy of the ball–Earth system remains constant, show that the tension in the string at the bottom is greater than the tension at the top by six times the weight of the ball.

72. A pendulum, comprising a string of length L and a small sphere, swings in the vertical plane. The string hits a peg located a distance d below the point of suspension (Fig. P8.72). (a) Show that if the sphere is released from a height below that of the peg, it will return to this height after striking the peg. (b) Show that if the pendulum is released from the horizontal position ($\theta = 90°$) and is to swing in a complete circle centered on the peg, then the minimum value of d must be $3L/5$.

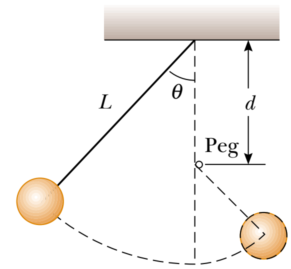
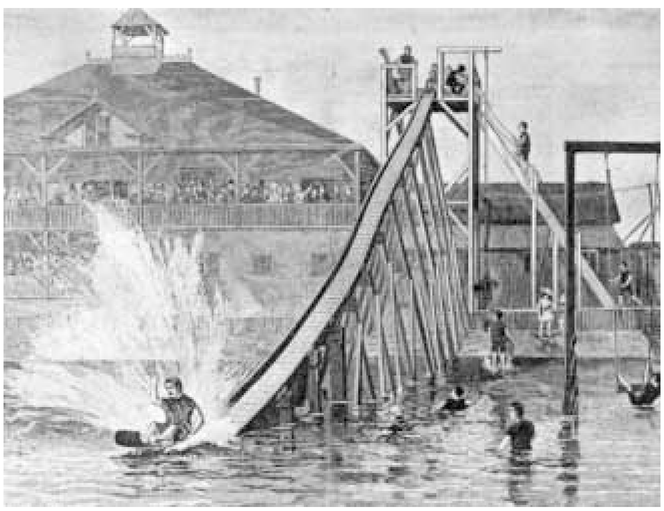


Figure P8.72

73. A roller-coaster car is released from rest at the top of the first rise and then moves freely with negligible friction. The roller coaster shown in Figure P8.73 has a circular loop of radius R in a vertical plane. (a) Suppose first that the car barely makes it around the loop: at the top of the loop the riders are upside down and feel weightless. Find the required height of the release point above the bottom of the loop in terms of R. (b) Now assume that the release point is at or above the minimum required height. Show that the normal force on the car at the bottom of the loop exceeds the normal force at the top of the loop by six times the weight of the car. The normal force on each rider follows the same rule. Such a large normal force is dangerous and very uncomfortable for the riders. Roller coasters are therefore not built with circular loops in vertical planes. Figure P6.20 and the photograph on page 157 show two actual designs.

Figure P8.73

74. **Review problem.** In 1887 in Bridgeport, Connecticut, C. J. Belknap built the water slide shown in Figure P8.74. A rider on a small sled, of total mass 80.0 kg, pushed off to start at the top of the slide (point Ⓐ) with a speed of 2.50 m/s. The chute was 9.76 m high at the top, 54.3 m long, and 0.51 m wide. Along its length, 725 wheels made friction negligible. Upon leaving the chute horizontally at its bottom end (point Ⓒ), the rider skimmed across the water of Long Island Sound for as much as 50 m, "skipping along like a flat pebble," before at last coming to rest and swimming ashore, pulling his sled after him. According to *Scientific American*, "The facial expression of novices taking their first adventurous slide is quite remarkable, and the sensations felt are correspondingly novel and peculiar." (a) Find the speed of the sled and rider at point Ⓒ. (b) Model the force of water friction as a constant retarding force acting on a particle. Find the work done by water friction in stopping the sled and rider. (c) Find the magnitude of the force the water exerts on the sled. (d) Find the magnitude of the force the chute exerts on the sled at point Ⓑ. (e) At point Ⓒ the chute is horizontal but curving in the vertical plane. Assume its radius of curvature is 20.0 m. Find the force the chute exerts on the sled at point Ⓒ.



Engraving from Scientific American, July 1888

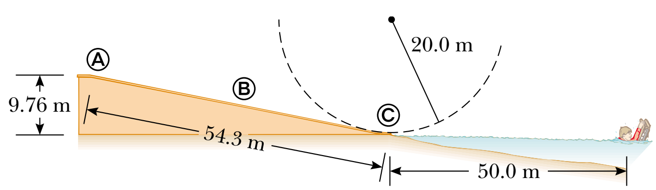


Figure P8.74

Answers to Quick Quizzes

8.1 (c). The sign of the gravitational potential energy depends on your choice of zero configuration. If the two objects in the system are closer together than in the zero configuration, the potential energy is negative. If they are farther apart, the potential energy is positive.

8.2 (c). The reason that we can ignore the kinetic energy of the massive Earth is that this kinetic energy is so small as to be essentially zero.

8.3 (a). We must include the Earth if we are going to work with gravitational potential energy.

8.4 (c). The total mechanical energy, kinetic plus potential, is conserved.

8.5 (a). The more massive rock has twice as much gravitational potential energy associated with it compared to the lighter rock. Because mechanical energy of an isolated system is conserved, the more massive rock will arrive at the ground with twice as much kinetic energy as the lighter rock.

8.6 $v_1 = v_2 = v_3$. The first and third balls speed up after they are thrown, while the second ball initially slows down but then speeds up after reaching its peak. The paths of all three balls are parabolas, and the balls take different times to reach the ground because they have different initial velocities. However, all three balls have the same speed at the moment they hit the ground because all start with the same kinetic energy and the ball–Earth system undergoes the same change in gravitational potential energy in all three cases.

8.7 (c). This system exhibits changes in kinetic energy as well as in both types of potential energy.

8.8 (a). Because the Earth is not included in the system, there is no gravitational potential energy associated with the system.

8.9 (c). The friction force must transform four times as much mechanical energy into internal energy if the speed is doubled, because kinetic energy depends on the square of the speed. Thus, the force must act over four times the distance.

8.10 (c). The decrease in mechanical energy of the system is $f_k d$, where d is the distance the block moves along the incline. While the force of kinetic friction remains the same, the distance d is smaller because a component of the gravitational force is pulling on the block in the direction opposite to its velocity.

8.11 (d). The slope of a $U(x)$-versus-x graph is by definition $dU(x)/dx$. From Equation 8.18, we see that this expression is equal to the negative of the x component of the conservative force acting on an object that is part of the system.